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Public debt is always toxic to economic growth

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The Aboa Centre for Economics is a joint initiative of the economics departments of the University of Turku and Åbo Akademi University.
This paper deals with the debt-growth relationship using several time-series tools. The idea is to find out whether the inverse relationship between these variable can be detected without imposing any functional forms for the estimating relationship and whether the relationship does indeed reflect some nonlinear features. Thus recursive correlations with different orderings of the time-series are computed using the Reinhart & Rogoff panel data. After that, recursive correlations are re-estimated with data that are cyclically adjusted to reflect the structural features of these two variables. The nature of the relationship is also scrutinized by using various variable-parameter estimation techniques (Kalman Filter, Logistic functional form and recursive estimation). Finally, some analyses of causality are carried out using these (filtered) data. The analysis clearly shows that the inverse relationship is very robust indeed and it rather supports the “toxic debt” hypothesis than the cyclical debt accumulation hypothesis.

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1. Introduction

The burden of public debt has long been a controversial issue in economics, as is shown by e.g. the classical review article of Tobin (1965). More recently, the relevance of this issue has increased dramatically because of the rapid acceleration of debt growth in practically all countries. The topic has also become the subject of intensive research, reflected in the rapidly increasing number of papers that have been published. In particular, the paper by Reinhart and Rogoff (2010) has been frequently cited, probably because they found very large negative growth effects. The specific interesting feature in Reinhart and Rogoff (2010) was their finding of a 90% debt-to-GDP threshold. Reinhart and Rogoff argue that if public debt exceeds this level, the growth rate slows dramatically. This finding was recently challenged by Herndon et al. (2013), who showed that the result was largely based on a computational error. The ‘computational error’ debate has recently dominated public media almost to the extent that the main issue of whether public debt hinders economic growth has been forgotten.

Why, then, should public debt have important negative growth effects? Several explanations can be put forward (see e.g. Boskin (2012)), ranging from crowding-out effects via higher interest rates to the income effects of higher debt service costs, increased uncertainty due to unanticipated financial market reactions to higher debt, loss of room for manoeuvre in monetary and fiscal policies due to the higher debt burden and so on. If we consider these arguments, it is hard to conceive that there may indeed be a universal fixed threshold for negative growth-rate effects. Rather, we might expect that a negative growth-rate effect could be discerned for all values of the debt-to-GDP ratio.

This is, in fact, the point of departure for our analysis. We do not intend to estimate any threshold for this relationship. Nor do we intend to solve the issue of causality between debt and GDP growth. We only scrutinize the debt GDP-growth relationship with the intention of establishing whether this relationship is indeed negative and whether it displays any signs of discontinuities. Hence we carry out a recursive computation of GDP growth and debt-to-GDP average values (increasing the sample by one observation at time from smallest to largest) for the Reinhart and Rogoff data. In addition to this simple correlation/regression analysis we carry out two a bit more sophisticated analysis of relationships between growth and indebtedness. We estimate the relationship by using the Kalman Filter and thus allowing the regression coefficient to change over the debt. Alternatively, we estimate the relationship in a logistic form thus allowing for more flexible functional relationship for the relevant
coefficients. We use recursive estimation so see whether relevant debt/GDP elasticity is indeed constant over time. Although we are well aware that we cannot solve the causality issue we compute a set of (Granger) causality statistics to find out how ambiguous the issue is. We try to shed some light to the causality issue by scrutinizing the relationship between investment (both private and public) and the Debt/GDP ratio in section 3. As for the data, we use two samples: one for the 19 advanced economies and another for all 64 countries in our data for 1950–2008. In the first sample we have 1,102 data points and in the second we have 5,539 data points. The investment data come from AMECO database and cover the period 1960-2012.

2. Results

Our results are reported in Figures 1–3. In Figure 1, we show the average values for 19 advanced economies, in the second those for all 64 countries in our data. In both cases we make recursive calculations starting from the smallest debt levels. Because that would give the high-debt countries relatively little weight, we also reverse the way of computing the average values, starting from the highest values of debt (Figure 3). The data are the same as Figure 1. The results of these calculations are strikingly robust for the different ways of computing the recursive values: there is always a negative slope in the debt-to-growth relationships. The growth rate loss may be as large as one percentage point, which is a shockingly high figure. Qualitatively, the shape of the relationship between the debt ratio and growth is similar to that obtained by Herndon et al. (2013) using locally smoothed regression, but we could argue that the recursive mean values are more sensitive to different segments of the data and less akin to the choice of the smoothness parameters. Clearly, there is no single threshold that would change the pattern of slower growth with higher debt. Contrary to Reinhart and Rogoff (2010)’s proposition that growth problems arise only when very high levels of debt are reached, the graphs suggest that a moderate debt burden may already seriously hinder economic growth. This, of course, depends on institutions, policy credibility and so forth, but, even so, we argue that there is no harmless level of public debt. Some support to the nonlinear effect could, however, be obtained by using Kalman Filter technique in estimating the bivariate relationship between GDP growth and debt/GDP relationship:
where $g$ denotes the growth rate of GDP and $D$ the (log of) Debt/GDP ratio. This simple model was estimated for the whole period 1820-2010 which includes 5539 observations that were ordered in an ascending order for all years and countries. The estimate of parameter $g_0$ was 5.787 (t-ratio 20.48) and the Log likelihood -16561. The final value of the state variable $\rho$ turned out to be -.685 (21.69). It appears that there is indeed some sort of kink in the relationship between these two variables but one has to be cautious in arguing on the behalf of a nonlinear impulse response. The scale differences are indeed rather small and they probably reflect the initial values of the coefficient $\rho$ in the subsample where the debt/GDP ratio is practically zero.

As a final check of the functional form, we estimated the bivariate relationship in the following logistic form

$$\Delta y_t = \alpha + \beta \left( \frac{1}{1 + \exp(\theta D_t/Y_t))} \right) + \tau D_t/Y_t$$  \hspace{1cm} (2)$$

where $\Delta y$ denotes GDP growth and $D/Y$ the Debt/GDP ratio. Here parameter $\theta$ controls the curvature of the relationship. The estimates are reported in Table 2 and the fitted values in Figure 6. Quite clearly the relationship resembles the one obtained by the Kalman Filter model (1). Thus, with very small debt levels GDP growth is, *ceteris paribus*, relatively high but with high and very level of debt the growth rate of GDP settled down to something like two per cent growth rate.

Of course, the plotted values (of correlation) can be interpreted in different ways from a causal point of view and our simple techniques cannot really solve the problem, although we may point out that the use of a lagged debt-to-GDP ratio instead of its contemporaneous value does not make any difference to the shape of the curvatures. The problem is that debt-to-GDP ratios are highly persistent, so that different lead-lag relationships are not very informative in solving the problem of causality. The conventional causality tests (Table 3) that we have carried out would suggest that causality is bi-directional, so that it would require an extreme view to argue that causality runs only from output growth to debt. Moreover, it is hard to

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1 For the point of reference, see the estimates of a simple linear model in Table 1.
imagine that a temporary slowdown in economic growth could produce this kind of (almost) monotonically increasing debt level.

If we consider a typical public balance reaction to a macroeconomic (output) shock we may write the following equation for the public sector balance:

\[
\text{Def}_t / Y_t = a + b\Delta y_t,
\]

where Def denotes the public sector balance (positive values surplus and negative deficit). The coefficient of \(\Delta y\) reflects the impact of income changes on taxes and expenditures (due to both automatic stabilizers and eventual counter-cyclical policy rules). The value of coefficient \(b\) could be thought to be something like 0.5 but the main thing is that the kind of relationship would imply a linear relationship between output growth and the change of debt/income level, not a relationship between the output growth and the level of debt. In fact, the fiscal rule relationship would not say anything about the level of debt assuming both automatic stabilizers and policy rules are related to cyclical movements of output. Only if we had a situation where the fiscal authority reacts to permanent slowdown of economic growth (which the authority falsely interprets a temporary demand shock) by progressively increasing public expenditures we would observe a negative relationship between economic growth and the level of debt. This may become evident if we eliminate entirely cyclical variations from the data. In drawing Figure 4, we have used 10-year moving averages of GDP growth and debt-to-GDP instead of annual rates in computing the recursive mean values. Clearly, the negative slope remains the same.

1. Debt and investment

Comparing GDP growth and government indebtedness is not necessarily very informative because the growth of GDP components may show completely different character. Take a simple example. GDP growth and indebtedness may be positively correlated because public consumption is financed by accumulating debt. A reduction of public consumption may produce an inverse result. A useful check would be to compare the investment share of GDP

\[\Delta (D_t / Y_t) = \text{Def}_t\]

That is because of the debt accumulation identity: \(D_t / Y_t = (1+r_t-\Delta y_t)D_{t-1}/y_{t-1} + \text{Def}_t\), which comes close to the identity \(\Delta(D_t / Y_t) = \text{Def}_t\). Buti and Sapir (1998) considered the sensitivity of government deficit with respect to output gap \(e = d(\text{Def}_t/Y_t)/d(y_t-\bar{y})\) and ended up with estimate of the magnitude of 0.5 for \(e\). See Bénassy-Quéré et al (2010) for more extensive treatment of this issue.

3 This kind of policies were indeed pursued in some countries in the 1970s after the first oil shock but it is difficult to find other instances of similar kind.
(or the share of public investment out of total public expenditures). If debt depresses investment we would obviously see a negative relationship between the investment share and the government indebtedness. If causality runs from investment to debt we should interpret the evidence as suggesting that negative investment shocks cause some financing problems within the government. The bigger is the investment shock, the more debt increases. Alternatively we could interpret the evidence from the point of structural problems: investment slowdown (that is caused by some third variables) causes a slowdown of GDP growth. If government tries to maintain the previous expenditure level it is only possible by continuously accumulating debt. According to recent paper by Salotti and Trecroci (2012) empirical evidence favors the interpretation that debts causes the slowdown of investment (and productivity) which obviously makes better sense for the trendlike movements of these two time series.

It has sometimes argued that debt accumulation is cased government’s additional expenditure to public investment – possibly as a reduction to private investment. It would be relatively easy to examine whether this conjecture is right by comparing these to time series. That is done in Figure 8, were we first compare the share of (private) investment of GDP and the Debt/GDP ratio (the recursive mean values is computed in the same way as in Figure 1-4). In the lower panel, we compare the share of public investment of total government expenditures. The outcome is quite clear: investment shares decrease almost monotonically along with the Debt/GDP ratio. It is easy to reconcile this outcome from the point of view of the toxic debt hypothesis but if we consider the reverse causality alternative we have rather adopt the structural view which says that governments react to investment slowdown by continuously accumulating debt. As for the public investment share, the interpretation is less ambiguous: increase in debt is clearly not caused by new public investment projects. Rather the borrowed money is used to transfers and public consumption. When there is no debt, the share of public investment of all expenditures is above 13 per cent but when debt goes to extreme levels (say, over 100 per cent of GDP), the share drops roughly to 7 per cent. A couple of decades ago David Aschauer (1989) showed that a slowdown of public investment could be one of the major reasons why economic growth had decreased dramatically since the 1970s. If his estimates are indeed applicable we could argue that the adverse development of public investment would indeed have contributed to slowdown of economic growth especially because it seems that no offsetting change has occurred with private investment.
2. Discussion

There is no doubt that there is an inverse relationship between debt and growth but the economic implications of this finding are not equally clear because the causality issue cannot be solved so easily. As pointed out earlier, we tend to favor the interpretation the excessive debt causes slowdown of growth rather than the opposite. Partly this is because the relationship not only shows up in actual data but also in cyclically adjusted data. Moreover, if (cyclical) income shocks caused the debt problems we would perhaps see the effect with the low levels of debt but not so much with the very high values of debt. In other words we would expect the debt/GDP -ratio elasticity to fall. But it seems that the relationship is rather linear than U-shaped\(^4\). The behavior of investment, particularly public investment, is also something that rather poorly corresponds to the explanation that the increase of debt is just of results of fiscal policies. Of course, these findings do not exclude the possibility that both causal explanations are resent at the same time. Rather, the question is of the relative (quantitative) importance of these explanations.

\(^4\) This also shows up in recursive estimates the log linear equation reported in Figure 7 (the corresponding OLS estimates are reported in Table 1 (on the second row).
References


Table 1  Estimation results of a simple linear bivariate model

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<tr>
<td>constant</td>
<td>4.249</td>
<td>(44.18)</td>
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<td>Debt/GDP</td>
<td>-.014</td>
<td>(10.06)</td>
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<td>R2 = 0.017, SEE = 4.807, Log likelihood = -16555</td>
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<tr>
<td>constant</td>
<td>5.898</td>
<td>24.97</td>
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<td>log(Debt/GDP)</td>
<td>-.703</td>
<td>10.64</td>
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<td>R2 = 0.020, SEE = 4.801, Log likelihood = -16549</td>
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The dependent variable is the growth rate of GDP. The sample period is 1820-2010.

Table 2  Estimates of a logistic regression model

\[
\Delta y_t = 1.899 - 5.415 \times (1/(1+\exp(-0.020\times D_t/Y_t)))
\]
\[t_1 = 4.96, t_2 = 7.62, t_3 = 3.89, R^2 = 0.020, SEE = 4.802, Log likelihood = -16549\]

\[
\Delta y_t = 6.400 - 2.645 \times (1/(1+\exp(-0.009\times D_t/Y_t))) - 0.011\times D_t/Y_t
\]
\[t_1 = 9.90, t_2 = 3.78, t_3 = 2.31, t_4=4.32, R^2 = 0.021, SEE = 4.801, Log likelihood = -16547\]

\[\Delta y = GDP\ growth, D/Y = Debt/GDP\]

Table 3  Pairwise Granger Causality Tests

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<th>Prob.</th>
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<td>D(DEBT) does not Granger Cause D(GDP)</td>
<td>5196</td>
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<td>D(GDP) does not Granger Cause D(DEBT)</td>
<td>12.44</td>
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<table>
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<th>F- Statistic</th>
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<td>Null Hypothesis:</td>
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<tr>
<td>D(DEBT) does not Granger Cause D(GDP)</td>
<td>2969</td>
<td>16.19</td>
<td>0.00</td>
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<tr>
<td>D(GDP) does not Granger Cause D(DEBT)</td>
<td>4.16</td>
<td>0.02</td>
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**Figure 1** Debt-to-GDP ratio and GDP growth in advanced economies

**Figure 2** Debt-to-GDP ratio and GDP growth for all countries

**Figure 3** Debt-to-GDP ratio and GDP growth from largest to smallest observation
Figure 4 10-year moving average of GDP growth and Debt-to-GDP ratio

Figure 5 Kalman filter estimates of the GDP coefficient and the fitted value GDP growth
**Figure 6**  Impact of Debt on GDP growth according to the logistic regression

In the upper panel, coefficient of linear term \( \tau \) is set to zero.

**Figure 7**  Recursive estimates of the log linear equation

Recursive estimates forward ± 2 S.E.
In the lower panel, the estimation is carried out by starting from the highest debt/GDP ratios.

*Figure 8  Debt and investment*

In the upper panel, there is the relationship between private investment/GDP and the Debt/GDP ratio, in the lower panel the share of public investment of total public expenditure and the Debt/GDP ratio.
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