Johan Willner
Public Options and Altruistic Firms - Antitrust Targets or Tools? The Welfare Impact of a Mixed Oligopoly With Managerial firms

Aboa Centre for Economics

Discussion Paper No. 59
Turku 2010
Johan Willner

Public Options and Altruistic Firms - Antitrust Targets or Tools? The Welfare Impact of a Mixed Oligopoly With Managerial firms

Aboa Centre for Economics
Discussion Paper No. 59
September 2010

ABSTRACT

I analyse the welfare impact of a mixed market with a public or private firm with some degree of altruism, in the presence of an agency problem. Contrary to some earlier findings, the total surplus turns out to be increasing in the degree of altruism. This impact is stronger than if there is no agency problem, despite more stringent conditions for the market to remain mixed. The altruistic firm is more cost-efficient, and viable if the market can remain mixed. A competition policy that encourages entry may increase welfare, but its scope is reduced by higher altruism.

JEL Classification: L32, L33, L44, H42
Keywords: non-profit maximising firms, public firms, mixed oligopoly, competition policy
Contact information

Johan.willner@abo.fi

Acknowledgements

The first version of this paper was prepared when I was visiting the Department of Economics, University of Warwick. I am grateful for its hospitality and for constructive response by participants in its Staff Workshop. I am also indebted to participants in the annual conference of the European Network on Industrial Policy, Reus, June 9th-11th, 2010, in particular to my discussant, Mehdi Feizi. The research is funded by an Academy Finland Research Grant (130986), as part of the Academy of Finland project Reforming Markets and Organisations (115003). Sonja Grönblom (in the same project) has also provided helpful comments. I am solely responsible for conclusions and remaining errors.
1. Introduction

A mixed oligopoly is a market where firms with different objectives interact. Most of the literature has focused on public firms with wider objectives that improve the market allocation, but the nonprofit firms can also be private. For example, Lakdawalla and Philipson (2006) and Philipson and Posner (2009) deal with the impact of altruism and the need for antitrust policies in its presence, and hence with a mixed market.

Historical examples of mixed oligopolies include markets with state ownership with wider objectives in the UK (Rees, 1984: 219n; Vickers and Yarrow, 1988: 127, 130-34; De Fraja 1991), Argentine (Xu and Birch, 1999), Finland (Miettinen, 2000), and to some extent the U.S. (Martin, 1959) and France (Sheahan, 1966). Private-sector examples include retail co-ops in Finland and Sweden (Willner, 2006b) and providers of health-care and education in the U.S. (Philipson and Posner, 2009). Despite being abortive, the proposed public option within the US health-care reform (see Krugman, 2010) shows at least that the theory of the mixed oligopoly is connected to real-life problems. Also, nationalisation (as a response to the present economic crisis) and liberalisation (of former public monopolies) have introduced mixed ownership in new markets (see De Fraja, 2009), and hence at least an opportunity for wider objectives.

Public firms became unfashionable during the era of privatisation, which was often seen as a way to improve cost efficiency. But the mixed oligopoly has been even more controversial when costs are not an issue, because of its threat to the survival of the profit maximisers (the Cournot-paradox; see Nett, 1993). For example, the US public option was feared to crowd out the private insurers (The Guardian, 29.9.2009). Low returns have in fact been described as subsidies that undermine private-sector profitability (Monsen and Walters, 1983: 120). Wider objectives are then seen as predatory behaviour and hence as an antitrust offence rather than a solution.¹ This is also related to the notion that altruism, in the form of wider objectives among private firms, is equivalent to a cost advantage (Lakdawalla and Philipson, 2006).

¹ The following definition by Cabral and Riordan (1997: 160) would classify nonprofit firms in mixed markets as predatory: "We call an action predatory if (1) a different action would increase the likelihood that rivals remain viable, and (2) the different action would be more profitable under the counterfactual hypothesis that the rival’s viability were unaffected." However, De Fraja (2009) suggests that such behaviour should be prosecuted only if it conflicts with the firm’s objective function.
A number of alternative assumptions have been adopted to ensure that an oligopoly remains mixed, but they also reduce its predicted benefits (Newbery, 2006). Profit maximisers would for example survive by being more efficient, but in many real-world markets this does not apply (see section 6). Increasing marginal costs would allow for survival among profit maximisers through lower marginal costs, but such an assumption is not supported by the empirical literature (Johnston, 1960; Martin, 2004). Exit would be prevented also through the more reasonable assumption that the altruistic firms combine social and commercial objectives. However, this would (in all previous models) limit the social benefits because of the imperfectly competitive restraint on output. Philipson and Posner (2009) suggest that both altruists and profit maximisers have an incentive to restrain output in this way. They argue that the deadweight loss is increasing in the degree of altruism and that antitrust legislation should therefore apply to all firms and markets.

While there is no crowding-out if marginal costs are increasing, the technical similarity between altruism and a cost advantage implies that the marginal firm is a profit-maximiser. It follows that altruism cannot improve the allocation if there is also free entry (Lakdawalla and Philipson, 2006). However, it is a stylised fact that entry is sluggish even in the presence of profit opportunities (Geroski, 1995). The case of mixed markets under conditions of imperfect competition and constant marginal costs might therefore deserve more attention.

The present contribution revisits the positive or negative welfare effect of a mixed oligopoly. I ask whether wider objectives or altruism are beneficial in the presence of an agency problem that affects cost-cutting incentives, and given a constraint that the profit-maximising firms should survive. Because of suggestions in the previous literature of a small or even negative welfare impact of a mixed oligopoly, I also analyse the need for a competition policy that encourages entry, as a complement or an alternative.

I set up a model with endogenous marginal costs that depend on the unobserved effort of a manager. Such models have been used before for analysing cost differences related to ownership and competition (De Fraja, 1993; Willner and Parker, 1997; Martin, 1993; Beiner et al., 2009). There is also a literature on incentive wages in a

---

2 In the case of public ownership, this is often called partial privatisation (Fershtman, 1990; Matsumara, 1998; Saha and Sensarama, 2007). However, a given share of state ownership does not necessarily translate to a similar weight for social welfare.

3 Some other approaches to keep an oligopoly mixed are described in Willner (2006a).
mixed-oligopoly. However, it can be shown that a complete alignment between the manager’s and the firm’s objectives is not necessarily optimal, and this literature has mainly focused on the strategic choice of weights in the reward function (Barros, 1995; Goering, 2007; Saha and Sensarama, 2008; Heywood and Ye, 2009). This is the first contribution to focus on potential cost distortions in a realistic mixed oligopoly with constant marginal costs and asymmetric information.

It turns out that the impact of altruism/wider objectives is positive and that it is strengthened when the endogeneity of the marginal costs is taken into consideration. The social costs of imperfect competition (if plausibly defined) are in other words decreasing in altruism, contrary to Philipson and Posner (2009). Also, the altruistic firm would survive without subsidies as long as the oligopoly remains mixed, i.e. as long as the private firms do not exit. The critical degree of altruism that would provoke exit is increasing in factors that increase the value of an agency parameter (the variance of the random component of the marginal costs and the manager’s risk-aversion and disutility of effort). Altruism reduces a firm’s marginal costs (but increases the marginal costs of its competitors). A policy that encourages entry can be a beneficial complement to the mixed oligopoly, but only if firms are few and if the degree of altruism is moderate. However, a public monopoly can otherwise be superior, in particular if the value of the agency parameter is low.

Next section presents the basic model, and section 3 a benchmark version with given marginal costs, showing that increased altruism always reduces the welfare loss of imperfect competition. Section 4 analyses the welfare effects in the presence of a principal-agent problem, and the scope for a policy that promotes entry. Section 5 extends the analysis to conditions where the public firm might get higher marginal costs. Section 6 presents concluding remarks and a discussion and interpretation of results that contradict conventional wisdom. Proofs are, when needed, included in an appendix.

2. The basic model

I analyse a market with $n+1$ firms and linear demand normalised as $p=a-x$, where $p$ and $x$ stand for price and output. Firm 1 maximises a weighted sum of its own profits and

\[\text{maximize } \frac{\alpha}{\alpha+1} \pi + \frac{1}{\alpha+1} \tau \]

4 For example, the manager of an altruistic firm can be given a stronger profit incentive so as to avoid an excessive output that would lead to high marginal costs.
industry output, as in Herr (2009). Such a model is simpler than if assuming a weight for the consumer surplus, but carries a similar logic as in a mixed oligopoly where one firm has wider objectives. This assumption makes sense in the light of the evidence of wider objectives than profit maximisation among state-owned firms (see section 1). However, this approach also corresponds to the analysis of private altruistic firms in Philipson and Posner (2009). A third and more sinister interpretation is that Firm 1 is a public firm that gives a weight to output because of politicians who need to please their voters (see Boycko et al, 1995). In what follows, we shall however refer to Firm 1 as ’altruistic’.

There are also \( n \) identical profit maximising or conventional firms; these will be indexed by \( i \). All firms are assumed to have Cournot-conjectures. Let \( \pi \) and \( \alpha \) stand for profits and the weight for output. The altruistic firm then maximises \( \alpha x + \pi_1 \), whereas a profit maximising firm \( i \) maximises just \( \pi_i \).

All firms are managerial. Firm 1 is managerial because most charities and state enterprises are led by an appointed manager. The profit-maximising oligopolists are managerial, because oligopolists tend to be relatively large and are rarely owner-managed. All managers are identical and they maximise utility, which is increasing in income and decreasing in effort. This rules out intrinsic motivation and a public-sector ethos.

There is asymmetric information in the sense that the owner cannot know whether for example high marginal costs are explained by bad luck or by a low effort. Marginal costs \( c \) depend on a constant parameter \( c_0 \), on the manager’s effort \( e \), and on a normally distributed random variable \( u \) with zero mean and the variance \( \sigma^2 \). Following Raith (2003) and Beiner et al. (2009), I assume that they can be written \( c = c_0 e + u \). The owner observes \( c \) and hence indirectly \( e + u \), but not its components. Also, the manager’s utility depends on income \( (w) \), disutility of effort \( (-ke^2/2) \), and a parameter \( (r) \) that can be shown to represent the Arrow-Pratt measure of absolute risk-aversion:

\[
U = -\exp[-r(w - ke^2/2)].
\] (2.1)
In order to ensure the existence of meaningful solutions, \( k > 1 \) must hold true.

With utility decreasing in effort everywhere, the manager would shirk without high-powered incentives. The employer therefore pays a performance-related wage that is increasing in the observable magnitude \( e + u \). Let \( w_0 \) denote a positive or negative intercept and \( \beta \) a coefficient that expresses the impact of the firm’s risky performance:

\[
w = w_0 + \beta (e + u).
\] (2.2)

The expected value and variance of the wage are then \( \bar{w} = w_0 + \beta e \) and \( \sigma_w^2 = \beta^2 \sigma^2 \). Substitute (2.2) for \( w \) in (2.1) and use the assumption of normally distributed shocks to rewrite the expected utility as a function of the expected income and its variance:

\[
V = w_0 + \beta e - \frac{r \beta^2 \sigma^2}{2} - \frac{ke^2}{2}.
\] (2.3)

Incentive compatibility means that the manager maximises her expected utility given the parameters of the reward function. Maximise (2.3) to get

\[
e = \frac{\beta}{k}.
\] (2.4)

With a zero reservation utility like in Beiner et al. (2009), the participation constraint becomes \( V = 0 \). Insert (2.4), solve for \( w_0 \) and insert \( w_0 \) and \( e \) into \( \bar{w} \):

\[
\bar{w} = \frac{1}{2} \left( \frac{\beta}{k} \right)^2 (r \sigma^2 k^2 + k).
\] (2.5)

The abbreviation

\[
\phi = r \sigma^2 k^2 + k
\] (2.6)

can be interpreted as a parameter that expresses the significance of the agency problem. It is larger than unity (because \( k > 1 \)) and increasing in disutility of effort, risk-aversion, and the variance of the random shocks. Use (2.5) and (2.6) to express the wage schedules in terms of this parameter:
Strictly speaking, the employer decides on $w_0$ and $\beta$, but we may as well use (2.7) and (2.8) to maximise the objective function directly with respect to $e$.

3. The impact of altruism in a mixed oligopoly with given marginal costs

This section provides a benchmark-model that enables us to compare the impact of altruism with and without endogenous marginal costs. It basically adds $n$ profit-maximising competitors to a monopoly model that is included in Philipson and Posner (2009). Suppose that the degree of altruism (i.e. the weight $\alpha$) is lower than under full welfare maximisation (see below), as for example when the authorities do not desire a public firm to become a monopolist. This can be interpreted as an imperfectly competitive restraint on output, so there is a social cost of preventing exit.

In this section marginal costs are given as $c$, so there is no principal-agent problem. The altruistic firm maximises

$$\Omega_1 = \alpha x_1 + ax_1 - xx_1 - cx_1,$$

whereas all other firms maximise

$$\pi_i = ax_i - xx_i - cx_i; \quad i = 2, 3, \ldots, n.$$ (3.2)

Let $x_p$ stand for $\Sigma x_i$. Because of the Cournot-conjectures, the first-order condition of Firm 1 can be written:

$$a - c + \alpha - 2x_1 - x_p = 0.$$ (3.3)

8 No results are affected by my normalisation that changes the slope of inverse demand from $-b$ to $-1.$
Rearrange the sum of the profit-maximising firms’ first-order conditions:

\[ n(a - c) - nx_1 - (n + 1)x_p = 0. \]  \hspace{1cm} (3.4)

Combining (3.3) and (3.4) yields the following output levels:

\[ x_1 = \frac{a - c + (n + 1)\alpha}{n + 2}, \]  \hspace{1cm} (3.5)

\[ x_p = \frac{n(a - c - \alpha)}{n + 2}, \]  \hspace{1cm} (3.6)

\[ x = x_1 + x_p = \frac{(n + 1)(a - c) + \alpha}{n + 2}. \]  \hspace{1cm} (3.7)

The price is then

\[ p = \frac{a + (n + 1)c - \alpha}{n + 2}. \]  \hspace{1cm} (3.8)

It is obvious that Firm 1 would be profitable for values of \( \alpha \) below \( a-c \) and that its profits are higher than in each profit-maximising firm.

If all firms maximise profits, we normally think of the deadweight loss as the difference between the imperfectly competitive total surplus and its first-best level \( TS^* \), associated with \( p^* = c \) and \( x^* = a-c \). However, such full welfare maximisation can only be achieved for \( \alpha = a-c \), in which case the private firms would exit, as follows from (3.7) and (3.6) respectively. The total surplus,

\[ TS(\alpha) = \frac{(n + 1)(n + 3)(a - c)^2 + 2(a - c)\alpha - \alpha^2}{2(n + 2)^2}, \]  \hspace{1cm} (3.9)

can therefore never reach its first-best value \( TS^* = (a-c)^2/2 \) without the market becoming a public monopoly. However, it can come arbitrarily close as \( \alpha \) approaches the upper bound of the open interval \( ]0,a-c[ \) in which the oligopoly can remain mixed. This justifies the use of \( TS^* \) as a point of comparison. The (absolute value of the) deadweight or welfare loss associated with a given value of \( \alpha \) is therefore:
It is obvious that (3.10) is decreasing and the total surplus increasing in $\alpha$.

These observations on the case of given and equal marginal costs are summarised in the following lemma, the proof of which is trivial and therefore omitted:

**Lemma 1.** i) The altruistic firm in a mixed oligopoly with given marginal costs is profitable if the private firms can survive, and its profits are higher than among its rival firms; ii) The deadweight loss of imperfect competition is decreasing in $\alpha$. iii) An increase in the number of firms when $\alpha$ is given increases welfare (reduces the deadweight loss).

In other words, the altruistic firm is viable in this context. The fact that more firms mean higher welfare is consistent Philipson and Posner (2009) who argue in favour of an antitrust policy that encourages entry also in the presence of altruism. However, part ii) contradicts their finding that the deadweight loss is increasing in $\alpha$, as implied by their interpretation of $\alpha$ as a cost advantage. The socially optimal monopoly price would then be $c-\alpha$, which would imply the deadweight loss

$$D(\alpha) = \frac{[a - (c - \alpha)]^2}{8}. \quad (3.11)$$

This expression is increasing in $\alpha$, for the same reason that it is decreasing in $c$. However, it is hard understand why the optimal allocation should require a monopolist to make losses if he is willing earn less than the maximum profits, when pure greed ($\alpha=0$) would allow him at least to break even.\(^9\) Also, why should a public firm be subsidised in the absence of other potential market failures? (The price cannot in addition equal $c-\alpha$ if $n>0$, because no profit-maximiser would then operate.)

\(^9\)By extension, consider a misanthropic monopolist who values a restraint of output as an end in itself ($\alpha < 0$). This would be formally equivalent to higher marginal costs. Philipson’s and Posner’s (2009) optimal price would then be $c+|\alpha|$, so their deadweight loss would be decreasing in $|\alpha|$.
A positive welfare impact may on the other hand be too small for being of any practical importance. Define the maximum impact of altruism as the percentage difference in total surplus between a conventional oligopoly (i.e., a market with \( n+1 \) firms and \( \alpha = 0 \)) and a mixed oligopoly with a total surplus close to but below \((a-c)^2/2\). Lemma 2 then makes it possible to compare the outcome in section 4 with the more conventional mixed oligopoly in this section:

**Lemma 2.** *The maximum percentage impact of altruism on a mixed market is*

\[
m_1 \approx 100 \frac{TS^* - TS}{TS} \bigg|_{\alpha = 0} = \frac{100}{(n + 1)(n + 3)}. \tag{3.12}
\]

**Proof:** See Appendix.

One, four and nine profit maximisers would for example correspond to values of \( m_1 \) of 12.50%, 2.86% and 0.83%, so altruism has a significant impact only if firms are few, like in the earlier literature.

**4. The case of endogenous marginal costs**

**4.1. Corporate performance**

The model in section 3 is now amended by assuming that the marginal costs \( c \) can be reduced through the manager’s effort, so that their expected size is \( c_0 - e \). There is now in addition a fixed cost, i.e. the manager’s wage \( \phi e^2/2 \). As follows from section 2, the expected values of the objective functions become:

\[
E\Omega_i = ax + ax_1 - xx_1 - (c_0 - e)x_1 - \frac{\phi}{2} e_1^2, \tag{4.1}
\]

\[
E\pi_i = ax_i - xx_i - (c_0 - e)x_i - \frac{\phi}{2} e_i^2. \tag{4.2}
\]

The mechanisms would be similar in a two-stage analysis, but it turns out to be simpler to deal with simultaneous maximisation. Maximise therefore with respect to \( x_1, e_1, \)
and \( e_i \), given the competitors’ choices. Impose thereafter ex post symmetry so that \( x_i = x_j \) and \( e_i = e_j \) for all \( i, j = 2,3,\ldots,n \) and rearrange. Use the inverse demand function to get \( p \) and note that \( \sum x_i = x_p \), \( x = x_1 + x_p \). We then get:

\[
\begin{align*}
  x_i &= \frac{(\phi - 1)(a - c_0) + [(n + 1)\phi - 1]\alpha}{(n + 2 - 1/\phi)(\phi - 1)}, \\
  x_p &= \frac{n[(a - c_0)(\phi - 1) - \phi\alpha]}{(n + 2 - 1/\phi)(\phi - 1)}, \\
  x &= \frac{(n + 1)(a - c_0) + \alpha}{n + 2 - 1/\phi}, \\
  e_i &= \frac{(\phi - 1)(a - c_0) + [(n + 1)\phi - 1]\alpha}{\phi(n + 2 - 1/\phi)(\phi - 1)}, \\
  e_p &= \frac{(a - c_0)(\phi - 1) - \phi\alpha}{\phi(n + 2 - 1/\phi)(\phi - 1)}, \\
  p &= \frac{(\phi - 1)a + \phi(n + 1)c_0 - \phi\alpha}{\phi(n + 2 - 1/\phi)}.
\end{align*}
\]

We can use this solution to make the following observations on the viability and comparative performance of the altruistic firm:

**Proposition 1.** i) The altruistic firm has lower expected marginal and average costs than its profit maximising rivals and an increase in its altruism increases their expected marginal and average costs; ii) An increase in the degree of altruism of Firm 1 reduces the weighted average of the expected marginal and average costs on the market; iii) The altruistic firm is profitable if \( \alpha \) is below the value \( \hat{\alpha} = (\phi - 1)(a - c_0)/\phi \) at which the private firms would exit.

**Proof:** See Appendix.

Part i) may seem controversial, but it extends results on efficiency and ownership in De Fraja (1993) and Willner and Parker (2007) to a mixed oligopoly. The intuition is based on the fact that wider objectives strengthen the incentive to buy efforts from an essentially lazy and greedy manager. However, section 5 extends the model so that another outcome becomes possible. As for the impact of increased altruism on rival’s costs, a higher \( \alpha \) reduces the profit-maximisers’ market share and profit margin, as follows from
(4.4) and (4.5). This makes it more difficult to afford paying for cost-cutting efforts. Part ii) means that this effect is not strong enough to lead to lower cost efficiency on average.

The intuition is again based on the profit maximisers’ falling market share, which compensates for their higher marginal and average costs when calculating the average.

Part iii) is important not least because of concerns that altruistic (public) firms would not survive in an open economy where subsidies are ruled out (Haskel and Szymanski, 1992; Bös, 1993). The altruistic firm would make losses also in this model if \( \alpha \) is too high, but such values would exceed \( \hat{\alpha} \) and would therefore be inconsistent with the oligopoly being mixed. The condition for the oligopoly to remain mixed is on the other hand stronger than in section 3. For example, if \( \phi = 3 \), \( \hat{\alpha} \) is now reduced by a factor of 2/3.

4.2. Welfare effects

According to Lemma 1, an increase in altruism increases welfare when marginal costs are given. In this subsection I ask whether this generalises to managerial firms. If so, how does the percentage impact of altruism in a mixed oligopoly compare to the analysis in section 3? I also ask how an increase in the number of firms affects the solution in this case.

In addition to the stakeholders in section 3, there are now also paid managers in the model, but their payoff (the reservation utility) is normalised to zero. Note also that the inverse demand function implies the consumer surplus \( x^2/2 \). The total surplus is therefore \( \pi_1 + \sum \pi_i + x^2/2 \):

\[
TS(\alpha) = \frac{(n + 1)\phi[\phi(n + 3) - 1] (a - c_0)^2}{2[(n + 2)\phi - 1]^2} + \frac{2\phi^2(a - c_0)}{2[(n + 2)\phi - 1]^2} \alpha
\]

\[
- \frac{\phi[(\phi - 1)^3 - (3n + n^2)\phi^2 + 2n\phi]}{2[(n + 2)\phi - 1]^2 (\phi - 1)^2} \alpha^2. \tag{4.9}
\]

A market with only profit maximisers would reach the total surplus \( TS(0) \), which is equal to the first term above. But if one of the firms is altruistic, the maximum total surplus is the limit of (4.9) as \( \alpha \) approaches \( (\phi - 1)/(\phi(a-c_0)) \). The highest possible impact of altruism is therefore

\[
m_2 \approx 100 \frac{TS^* - TS(0)}{TS(0)} = 100 \frac{\phi^3 + (n^2 + 3n + 1)\phi^2 - (3 + 2n)\phi + 1}{(n^2 + 4n + 3)\phi^3 - (n + 1)\phi^2}. \tag{4.10}
\]
As it turns out, the welfare effect of altruism is positive like in the previous section, provided that the oligopoly can remain mixed, and stronger than in section 3:

**Proposition 2.** i) An increase in the degree of altruism of Firm 1 increases the total surplus; ii) The maximum percentage impact of altruism is higher than when marginal costs are given.

**Proof:** See appendix.

Part i) differs from Philipson and Posner (2009) because of their definition of the deadweight loss. It also differs from Heywood’s and Ye’s (2009) result that a mixed oligopoly with \(n>1\) may reduce welfare, because they assume increasing marginal costs.\(^{10}\) However, the intuition behind i) and ii) is fairly obvious in the light of Proposition 1, because of the reductions of both costs and the profit margin. Part ii) is important given the limited or negative social benefits in the earlier literature, and may seem surprising because of the now lower value of \(\hat{\alpha}\). For example, \(m_1\) in section 3 is only 0.83% if there are nine firms, but now we get \(m_2 = 43.8\) if \(\phi=2\) and \(m_2 = 22.34\) if \(\phi=5\).

Despite this powerful impact, we may ask whether it would always be beneficial that profit maximising firms enter. Suppose first that all firms maximise profits (\(\alpha=0\)). As follows from (4.7), increased competition would then reduce the ability to afford cost-reducing efforts, so marginal costs would in fact increase.\(^{11}\) Entry can still be beneficial, but only up to a point:

**Lemma 3.** i) An increase in the number of firms on a market with managerial profit-maximising firms increases welfare if there are fewer firms than \(\hat{n} = 2\phi - 4 + 1/\phi\) and vice versa; ii) The critical value \(\hat{n}(0)\) is increasing in risk and in the managers’ risk-aversion and disutility of effort.

\(^{10}\)A welfare reduction may follow from also higher public-sector wages (Herr, 2009), e.g. because of bargaining (but the excess wages are then part of the total surplus; see Grönblom and Willner, 2008). However, the empirical evidence is inconclusive. In developed countries, females tend to get relatively higher and males lower wages (Melly, 2005; Ramoni-Perazzi, 2006; Disney, 2007; Disney and Gosling, 2008). This may also be consistent with private rather than public-sector wage distortions.

\(^{11}\)This reflects earlier results on the relationship between cost efficiency and competition in Cournot-models with managerial firms (see Martin, 1993; Willner and Parker, 2007).
Proof: Se appendix.

It is well-known that too many firms would enter under conditions of free entry and non-zero sunk costs (Mankiw and Whinston, 1986). However, there are no sunk costs in this model. The normalisation of the managers’ outside-option utility to zero means that there is no upper bound for \( n \), as implied by (4.4). The welfare loss when \( n > \hat{n} \) is therefore not caused by duplication of sunk costs but by higher marginal costs that overshadow the benefits of lower profit margins. Values of \( \phi \) below 1.71 can even imply that a monopoly is welfare superior. However, a more severe agency problem may paradoxically increase the scope for welfare-increasing competition. For example, \( \phi = 3 \) would imply \( \hat{n} = 3 \) (because \( \hat{n} \) is the integer closest to 2.33), but \( \phi = 10 \) would suggest \( \hat{n} = 16 \).

The intuition for the impact of \( \phi \) is based on the fact that the manager’s equilibrium salary is decreasing in \( \phi \) (despite being increasing for a given effort), as follows from inserting (4.6) and (4.7) into (2.7) and (2.8) when \( \alpha = 0 \). When \( \phi \) becomes very large, marginal costs approach \( c_0 \) and the salary zero, so we approach a conventional market with no cost reducing efforts and hence no managerial salary. As long as \( a > c_0 \), more firms would then always mean a higher total surplus, like in a conventional model.

The following holds true when one of the firms is altruistic:

Proposition 3. The optimal number of profit maximising firms in a mixed market, 

\[
\hat{n} (\alpha) = \frac{(2\phi^2 - 4\phi + 1)(\phi - 1)(a - c_0) - (2\phi^2 - 1)\phi\alpha}{\phi(\phi - 1)(a - c_0) + \phi\alpha},
\]  

(4.11)

is decreasing in the degree of altruism.

Proof: Se appendix.

In other words, altruism reduces the scope for entry to improve welfare. This suggests that it is not always necessary to apply a competition policy that encourages entry an
markets with altruistic firms, contrary to Philipson and Posner (2009).\(^{12}\)

In fact, a public monopoly may be better than the mixed oligopoly even if \(\alpha\) is low enough to permit profit-maximising competitors to operate. This would be the case if

\[
\hat{\alpha} \geq \alpha \geq \frac{2\phi^2 - 4\phi + 1}{2\phi^2 - 1} \hat{\alpha},
\]

(4.12)

because \(\hat{n}\) is then zero or negative. This extends the result that the optimal market may be a public monopoly if \(\alpha = 0\) and if \(\phi\) is small (Lemma 3) to the case of non-zero values of \(\alpha\) and low values of \(\phi\). The intuition is the fact that higher values of \(\alpha\) makes it more difficult for the profit maximisers to afford cost-cutting efforts. When \(\alpha\) gets sufficiently high, society would be better off by exit.

On the other hand, the best market structure includes one or more profit maximising firm if

\[
0 < \alpha < \frac{2\phi^2 - 5\phi + 1}{2\phi^2} \hat{\alpha},
\]

(4.13)

because \(\hat{n}(0) \geq 1\) then holds true.\(^{13}\)

For example, setting \(\phi\) equal to 2, 3 and 10 would imply values of \(\hat{\alpha}\) of 0.50(\(a-c_0\)), 0.67(\(a-c_0\)) and 0.90(\(a-c_0\)). A public monopoly would then be superior despite the possibility of coexistence for values of \(\alpha\) exceeding 0.07(\(a-c_0\)), 0.27(\(a-c_0\)) and 0.73(\(a-c_0\)). This suggests that higher risk, risk-aversion and disutility of effort would permit not only more altruism before the market becomes a monopoly, but also increase the threshold of altruism that makes a monopoly more desirable.

\(^{12}\)Their result is partially driven by increasing marginal costs, which mean that a restraint on output among altruists leads to higher marginal costs among the profit maximisers.

\(^{13}\)Note that this analysis is approximate, because \(n\) must be an integer. A direct comparison of the expressions for the total surplus according to (4.9) is necessary if (4.11) yields a value in the open interval ]0,1[.
The impact of $\phi$ on the desirability of welfare-increasing entry is more complicated than in Lemma 3. Set $a-c_0=1$ and suppose that $\alpha=0.25$. A public monopoly is then superior if $\phi=3$, whereas $\phi=5$ or $\phi=10$ would mean a welfare increase until there are two or ten firms respectively.\footnote{Using (4.11) would in fact yield 2.9529 for $\phi=5$, but it turns out that the total surplus is higher if there are two rather than three profit maximisers.} With $\alpha=0.5$, the monopoly is superior for $\phi=3$ and $\phi=5$, but $n=14$ if $\phi=10$. Thus, entry is not always desirable, contrary to section 3, but the scope for competition is increasing in the agency parameter under reasonable circumstances.

5. An extension to higher risk-aversion in the altruistic firm

The assumption that risk, risk-aversion, and disutility of effort are the same in Firm 1 and among its rivals has led to the conclusion that the altruistic firm is more cost efficient, with the controversial corollary that public firms with wider objectives are more efficient than private profit maximisers. However, assuming a higher $\phi$ in Firm 1 can - under some circumstances - yield another conclusion. This might be the case if production in the altruistic firm is more risky, or if it attracts more lazy or risk-averse managers. These alternatives are technically equivalent, but I refer for brevity to a higher $\phi$ as reflecting higher risk-aversion. Let the values of $\phi$ be denoted by $\phi_1$ and $\phi_p$ respectively. Suppose that $\phi_1>\phi_p$. This yields the following solutions:

\begin{align*}
x_1 &= \frac{\phi_1((\phi_p-1)(a-c_0) + [(n+1)\phi_p-1]\alpha)}{(\phi_1-1)[(n+1)\phi_p-1] + \phi_1(\phi_p-1)}, \quad (5.1) \\
x_p &= \frac{n\phi_p[(\phi_1-1)(a-c_0) - \phi_1\alpha]}{(\phi_1-1)[(n+1)\phi_p-1] + \phi_1(\phi_p-1)}, \quad (5.2) \\
x &= \frac{(\phi_1\phi_p - \phi_1 + n\phi_1\phi_p - n\phi_p)(a-c_0) + \phi_1(\phi_p-1)\alpha}{(\phi_1-1)[(n+1)\phi_p-1] + \phi_1(\phi_p-1)}, \quad (5.3) \\
e_1 &= \frac{(\phi_p-1)(a-c_0) + [(n+1)\phi_p-1]\alpha}{(\phi_1-1)[(n+1)\phi_p-1] + \phi_1(\phi_p-1)}, \quad (5.4) \\
e_p &= \frac{(\phi_1-1)(a-c_0) - \phi_1\alpha}{(\phi_1-1)[(n+1)\phi_p-1] + \phi_1(\phi_p-1)}, \quad (5.5)
\end{align*}
In the special case of $\alpha=0$, Firm 1 is also a profit maximiser, in which case $\phi_1 > \phi_p$ would imply $e_1 < e_p$. However, a nonzero $\alpha$ can in the opposite case compensate for a higher $\phi$. As follows from (5.2), the private firms would exit if $\alpha \geq (\phi_1 - 1)(a-c_0)/\phi_1$. With this restriction in mind, Firm 1 remains more cost efficient despite the fact that $\phi_1 > \phi_p$ if

$$\frac{\phi_1 - \phi_p}{(n + 1)\phi_p + \phi_1 - 1}(a-c_0) > \alpha > \frac{\phi_1 - 1}{\phi_1}(a-c_0).$$

(5.7)

Note that the interval above can never be empty, because $\phi_1$ and $\phi_p$ are larger than unity. Thus, while a mixed oligopoly where $e_1 < e_p$ is possible, $e_1 > e_p$ holds true if

$$\frac{\phi_1 - \phi_p}{(n + 1)\phi_p + \phi_1 - 1}(a-c_0) > \alpha.$$ 

(5.8)

We can conclude that the analysis in section 4 applies unless the difference $\phi_1 - \phi_p$ is too large, for example when a higher $r$ puts Firm 1 too much at a disadvantage. In such cases the beneficial effects of the mixed oligopoly become small or even negative.

We may also ask whether the viability of Firm 1 may become threatened. As follows from (5.1), $x_1$ is always positive. The public firm is therefore profitable if and only if $a-x-c_0+e_f/2$ is positive (because $e_f = x_1/\phi_1$). Note also that $\alpha < (\phi_1 - 1)(a-c_0)/\phi_1$ must hold true, because the oligopoly would not remain mixed otherwise. As follows from (5.1), (5.3) and (5.4), Firm 1 is then able to break even for any $\alpha$ that is consistent with a mixed oligopoly, despite the disadvantage implied by a higher value of $\phi$.

6. Discussion and concluding remarks

The earlier literature tends to suggest that the welfare impact of a mixed oligopoly is limited if positive at all. The present contribution means by contrast that the presence of a firm with wider objectives is beneficial, with a potentially strong impact when marginal costs are constant but dependent on unobservable managerial efforts. The altruistic firm breaks even whenever the profit-maximising firms are willing to participate, even if the
manager of the altruistic firm is more risk averse or lazy. For high values of a parameter that summarises risk, risk-aversion and the disutility of effort, there may be (a limited) scope for a traditional antitrust policy that encourages entry. However, the optimal number of firms is decreasing in the degree of altruism.

The most controversial finding may be related to the cost-efficiency of the public or private altruistic firm. It flies in the face of conventional wisdom, but the theoretical and empirical literature provides more mixed results than usually believed. While the surveys by Borcherding et al. (1982), Megginson and Netter (2001) and Dewenter and Malatesta (1997) do indeed suggest that public ownership reduces efficiency, a different and much less black-and-white picture emerges from Millward (1982), Boyd (1986), Iordanoglou (2001), Martin and Parker (1997), Willner (2001) and Florio (2004). Views are conflicting also among theorists (see Pint, 1991; Estrin and Perotin, 1991; De Fraja, 1993; Willner and Parker, 2007).

The fact that a combination of such familiar assumptions as Cournot-competition, coexistence between altruistic and profit-maximising firms and asymmetric information implies an outcome that is strongly in favour of a mixed oligopoly (also when associated with public ownership) is however interesting also as an observation on mainstream theory. The result on corporate performance partly follows from the fact that standard theory treats the managerial effort to reduce costs as a commodity with a price. Only firms with wider objectives than profit maximisation are then prepared to pay properly.

However, while it is too simplistic to dismiss the mixed oligopoly because of alleged cost inefficiency or crowding-out, the present analysis should be interpreted carefully. Real-world state-owned enterprises do not necessarily have objective functions such as in the model, and distorted objectives are possible also among private nonprofits (see Galaskiewicz and Bielefeld, 2003). Other modelling details may also be questioned. In particular, the analysis has ignored the possibility of efforts without external rewards and punishments, despite their potential significance in particular in public-sector and other not-for-profit organisations (Francois, 2000). A proper understanding of the challenges facing different types of firms and organisations on a mixed market might therefore require a more careful analysis of work motivation among both managers and other staff.

Appendix

Proof of Lemma 2. Use (3.5)-(3.8) to write the total surplus for the case $\alpha=0$ as
To get (3.12), calculate the percentage difference as compared to $TS^{*}$ as defined by (3.9) and simplify. QED.

**Proof of Proposition 1:** i) The expected marginal costs are $c_0\phi e_1$ and $c_0\phi ep$, and must therefore be lower in Firm 1 because of (4.6) and (4.7). As for expected average costs, they are $c_0\phi e_1+\frac{\phi e_1^2}{2}x_1 = c_0\phi e_1/2$ and $c_0\phi ep+\frac{\phi ep^2}{2}x_p = c_0\phi e_1/2$, as follows from (4.3)-(4.4) and (4.6)-(4.7) and therefore lower in the altruistic firm.

ii) Note that the weighted average $\bar{e} = (x_1 e_1 + x_p e_p) x$ is

$$\bar{e} = \frac{A + B\alpha + C\alpha^2}{(D + \alpha)E},$$

(A.2)

where

$$A = (n + 1) (\phi - 1)^2 (a - c_0)^2,$$

(A.3)

$$B = 2(\phi - 1)^2 (a - c_0),$$

(A.4)

$$C = \phi^2(n^2 + 3n + 1) - 2(n + 1)\phi + 1,$$

(A.5)

$$D = (n + 1) (a - c_0),$$

(A.6)

$$E = [(n + 2)\phi - 1] (\phi - 1)^2.$$  

(A.7)

Differentiating with respect to $\alpha$ and rearranging shows that the average effort is increasing in $\alpha$ (and the average marginal costs hence decreasing in $\alpha$) under the condition

$$BD - A + 2CD \alpha + C\alpha^2 > 0$$

(A.8)

and vice versa. Note that $C$ must be positive, because both roots of the equation $C=0$ are complex for $n>0$. It is also obvious that $BD > A$. The average marginal costs are therefore decreasing in $\alpha$. As for average costs, it follows from the prof of part i) that their weighted average must be decreasing in $\alpha$. Part ii) is thereby proved.
iii) It is obvious from (4.4) that none of the profit-maximising firms would produce if 
\[ \alpha > \alpha = (\phi - 1)(a - c_0)/\phi. \]
As for Firm 1, its profits are 
\[ (p - c_0 + e_1)x_1 - \phi e_1^2/2, \]
or
\[ \pi_1 = \left( \frac{(\phi - 1)(2\phi - 1)(a - c_0) - [2\phi^2 - (3 + n)\phi + 1]x_1}{2[(n + 2)\phi - 1] (\phi - 1)} \right)x_1. \]  
(A.9)

Firm 1’s output is never negative, as follows from (4.3). The sign of the expression 
therefore depends on the parenthesis to the left, which can be negative only if 
\[ \alpha > \frac{(\phi - 1)(2\phi - 1)(a - c_0)}{2\phi^2 - (3 + n)\phi + 1}. \]  
(A.10)

However, it is obvious that this boundary is beyond the limit \((\phi - 1)(a - c_0)/\phi\) at which the 
profit-maximising firms would exit. We can therefore conclude that the altruistic firm is 
viable for all values of \(\alpha\) that are compatible with a mixed oligopoly remaining mixed. Part 

iii) is thereby proved. QED

Proof of Proposition 2. i) Use (4.3)-(4.8) and add the consumer surplus \(x^2/2\) to the 
profits, which are 
\[ (p - c_0 + e_1)x_1 - \phi e_1^2/2 \]
and rearrange to get the total surplus as expressed by (4.9). It is obvious that its second term is positive for all \(\alpha > 0\). If 
the third term is also positive, it is trivially true that the total surplus is increasing in \(\alpha\). In 
the opposite case, \(TS\) is concave with a maximum for some value \(\alpha^M\) of \(\alpha\):

\[ \alpha^M = \frac{\phi(1 - \phi)^2(a - c_0)}{(\phi - 1)^3 - (3n + n^2)\phi^2 + 2n \phi}. \]  
(A.11)

However, \(TS\) cannot be locally decreasing unless \(\alpha^M < (\phi - 1)(a - c_0)/\phi\) (see Proposition 1). 
Suppose as an antithesis that this is the case:

\[ \frac{\phi(1 - \phi)^2(a - c_0)}{(\phi - 1)^3 - (3n + n^2)\phi^2 + 2n \phi} < \frac{(\phi - 1)(a - c_0)}{\phi}. \]  
(A.12)

This inequality can be satisfied only for the following values of \(\phi\):
It is obvious that this would require values of $\phi$ below unity, which is ruled out by our assumption that $k>1$. The total surplus is therefore increasing in $\alpha$ in the relevant interval. Part $i)$ is thereby proved.

$ii)$ Suppose as an antithesis that there exist strictly positive values of $n$ and values of $\phi$ such that $\phi > 1$ for which $m_1 > m_2$. Use Lemma 2 and (4.10) to rearrange the condition $m_1 > m_2$ to

$$(n^3 + 6n^2 + 10n + 4)\phi^2 - (3 + 2n)(n + 3)\phi + (n + 3) < 0. \quad (A.14)$$

Combine this with the inequality

$$1 - \phi < 0 \quad (A.15)$$

so as to get

$$(n^3 + 6n^2 + 10n + 4)\phi^2 - (2n^2 + 9n + 8)\phi + (n + 4) < 0. \quad (A.16)$$

Set the left hand side of (A.16) equal to zero so as to get a second-degree equation with the roots

$$\phi_{1,2} = \frac{2n^2 + 9n + 8}{2(n^3 + 6n^2 + 10n + 4)} \pm \sqrt{-\frac{-4n^3 - 23n^2 - 32n}{4(n^3 + 6n^2 + 10n + 4)^2}}. \quad (A.17)$$

These roots are complex unless $n=0$, which would mean a monopoly, not a mixed oligopoly. It follows that (A.16) is positive for all $\phi > 1$ and $n>0$. Hence, the maximum percentage impact of altruism is always larger in a model with a principal-agent problem.

QED

Proof of Lemma 3. $i)$ Differentiate (4.9) with respect to $n$ when $\alpha=0$ and set the derivative equal to zero:
Solving for \( n \) yields \( \hat{n}(0) \approx 2\phi - 4 + 1/\phi \). This is a maximum, because it is obvious that the derivative in question is decreasing in \( n \). Part \( i \) is thereby proved.

\( ii \) Differentiate \( \hat{n}(0) \) with respect to \( \phi \). It follows that \( \hat{n}(0) \) is minimised if \( \phi = \sqrt{2} \).

However, as this would imply a negative \( \hat{n}(0) \), it follows that \( \hat{n}(0) \) is increasing in \( \phi \) in the relevant area. Part \( ii \) is thereby proved. QED

**Proof of Proposition 3**: Differentiate (4.9) with respect to \( n \), rearrange and solve for an extreme value that has to be a maximum:

\[
\hat{n}(\alpha) \approx \frac{(\phi - 1)^2 (2\phi^2 - 4\phi + 1)(a - c_0)^2 - 4\phi^2 (\phi - 1)^2 (a - c_0)\alpha + \phi^2 (2\phi^2 - 1)\alpha^2}{\phi(\phi - 1)^2 (a - c_0)^2 - \phi^2 \alpha^2}.
\]  

(A.19)

However, \( \alpha = \hat{\alpha} = (\phi - 1)(a - c_0)/\phi \) satisfies the polynomials both in the numerator and denominator. Use this to simplify the expression to get (4.11). Note that its numerator is decreasing and its denominator is increasing in \( \alpha \), so \( \hat{n}(\alpha) \) must be decreasing. QED

**REFERENCES**


Martin, Stephen (2004), 'Globalization and the Natural Limits of Competition’ in Neumann, Manfred and Jürgen Weigand (Eds.), *Handbook of Competition*, Cheltenham: Edward Elgar.


Ramon-Perazzi, Josef and Bellante, Don (2006), 'Wage Differentials Between the Public and The Private Sector: How Comparable are The Workers?', *Journal of Business & Economics Research*, vol. 4, no. 5, pp. 43-57.


Aboa Centre for Economics (ACE) was founded in 1998 by the departments of economics at the Turku School of Economics, Åbo Akademi University and University of Turku. The aim of the Centre is to coordinate research and education related to economics in the three universities.

Contact information: Aboa Centre for Economics, Turku School of Economics, Rehtorinpellonkatu 3, 20500 Turku, Finland.

Aboa Centre for Economics (ACE) on Turun kolmen yliopiston vuonna 1998 perustama yhteistyöelin. Sen osapuolet ovat Turun kauppakorkeakoulun kansantaloustieteen oppiaine, Åbo Akademin national-ekonomi-oppiaine ja Turun yliopiston taloustieteen laitos. ACEn toiminta-ajatuksena on koordinoida kansantaloustieteen tutkimusta ja opetusta Turun kolmessa yliopistossa.

Yhteystiedot: Aboa Centre for Economics, Kansantaloustiede, Turun kauppakorkeakoulu, 20500 Turku.

www.ace-economics.fi

ISSN 1796-3133