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A Cluster and a Search Market Can Coexist

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ABSTRACT

I examine a rational expectations model of buyers and capacity constrained sellers, where traders can choose between a cluster and a search market. Sellers choose a market and post a price, and then buyers choose which market to visit. There is a pure strategy equilibrium where all agents are in the cluster. There are also two continua of mixed strategy equilibria. In a low-price equilibrium the cluster features overdemand, and in the high-price equilibrium the cluster features oversupply. The result differs sharply from a model where demand is stochastic and realizes after sellers have made their choices.

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1 Introduction

I study the coexistence of a cluster and a search market in a model of buyers and sellers. A cluster is a marketplace where sellers are located very close to each other, such that buyers can visit many sellers and compare their goods and prices at a negligible cost. In a search market sellers are dispersed, and buyers' can visit only few, maybe just one, seller in a period. For many types of goods both types of markets can be observed. For example, watches can be bought in stores that are located close to each other in a large shopping center, but they can be bought also from watchmakers located far from each other. Used cars are on offer in large car dealers' premises which are located next to each other, but they can be bought also from an individual owners who live all around a city.

Although a cluster offers the agents a large number of potential partners, the negative aspect of it is the increased congestion on an agent's own side. Sellers are exposed to tough competition by other sellers, and buyers have more rivals wishing to buy the same item. In a search market it is more difficult to meet trading partners, but both sides of the market enjoy some monopoly power. These tradeoffs may lead to a situation where there are sellers in a cluster as well as in a search market. Then some buyers would like to visit the cluster, whereas some buyers choose to go to the search market.

I consider a static rational expectations model where there is a large fixed number of sellers with a unit of an indivisible good, and a large fixed number of buyers with a unit demand. The numbers of buyers and sellers are common knowledge. Sellers choose either a cluster or a search market and post binding prices which are observed by all. I assume that in each market sellers use a symmetric strategy in pricing. Then buyers choose which market to go to. I assume that in the cluster there are no search frictions, and all the agents on the short side of the cluster trade. In the search market buyers observe the prices, and each buyer chooses a seller. Buyers' actions are uncoordinated, and because of symmetric pricing the search market is of an urn-ball type. I find that the model exhibits three types of equilibria: A pure strategy equilibrium where all sellers are in the cluster, and two mixed market equilibrium continua where there are buyers and sellers in both markets. There is a low-price equilibrium where sellers are on the short side of the cluster, and there is a high-price equilibrium where sellers are on the long side of the cluster.

Fisher and Harrington (1996) study the coexistence of a cluster and a search market. There is an endogenous number of sellers who each produce an indivisible piece of goods. They choose either a cluster or a search market. Sellers set prices in both markets. Then buyers choose an initial location. The cost of entering the cluster is random. Once in the cluster a buyer can sample all sellers at no cost. In the search market buyers sample one firm per period at the same cost per visit. When meeting a seller the type of goods (which is also the buyer's willingness to pay) is drawn from a uniform distribution. Buyers search with recall in both markets, and they can switch between the markets. The larger the heterogeneity the more sellers the buyers wish to meet, and more buyers go to the cluster. This increases sellers' incentive to choose the cluster. But the larger the heterogeneity the larger the local monopoly power of sellers in the search market, and more sellers choose the search market. The first effect dominates, and only the cluster survives, except if goods are very heterogeneous.

In Neeman and Vulkan (2010) goods are homogenous, but production costs and buyers' willingness to pay are stochastic. Agents cannot switch to another market within a period. The terms of trade are determined by bargaining in the search market and by Walrasian market-clearing price in the cluster. Only the cluster survives, because a trader can keep almost the entire match surplus he generates, whereas in the search market he has to share the surplus with his partner. The search market unravels because in each period the traders with relatively large potential gains switch to the cluster.

In Kultti (2011) there is a stochastic measure of buyers. There is a large and fixed measure of sellers who choose between a search market and a cluster and post a price. The measure of buyers is realized after sellers have made their choices. After the number of buyers is realized, the buyers observe the prices and choose between the two markets. Sellers' choices about locations and prices reflect their expectations of demand. In equilibrium the markets do not coexist: If buyers are indifferent between the markets, sellers fare better in the cluster. There are at most two equilibria: either all agents choose the cluster, or all agents choose the search market. Using a trembling-hand argument (where agents may make mistakes in evaluating the expected value of their choice) it is shown that only the cluster survives in equilibrium.

The rest of the paper is organized as follows: Section 2 presents the model. In Section 3 I consider a low-price mixed market equilibrium, and in section 4 I consider a high-price mixed market equilibrium. Section 5 is devoted to a stability analysis of mixed market equilibria. In Section 6 I solve the pure strategy equilibria. Section 7 concludes.

2 The Model

The economy has B buyers and S sellers, and B and S are large, fixed numbers. Let $\theta \equiv B/S$. Each seller has a nondivisible unit of a good which he values at zero. Each buyer wishes to buy one unit of the good which she values at one. If a seller sells the unit, his utility is equal to the price of the unit. If a buyer buys the unit, her utility is one minus the price.

There are two markets where trading can take place: a cluster and a search market. In both markets sellers post a price. In the cluster sellers are located close to each other such that there is no search friction. If two or more buyers happen to choose the same seller, all the buyers except one relocate themselves to other sellers. This continues until all potential buyer-seller pairs are formed. Then all agents on the short side of the cluster trade, and agents on the long side are rationed with equal probabilities.

In the search market each seller is in a location by himself and posts a price. The prices are observed by all agents, and buyers choose which seller to go based on the prices. Sellers' locations are dispersed such that having contacted a seller a buyer cannot relocate herself. Because of buyers' uncoordinated actions there may be local overdemand or oversupply: some sellers receive several buyers while some sellers do not receive any buyers. Therefore not all potential trades are made. Suppose for a while that sellers do not post prices, and buyers choose sellers randomly. Then the search market is of urn-ball type where sellers represent urns and buyers represent balls. The number of buyers a seller meets is binomially distributed. As the binomial distribution is quite cumbersome to deal with, I utilize the assumption that numbers of buyers and sellers are large, and I approximate the binomial distribution by a Poisson distribution. Then the probability that a seller does not meet any buyers is $e^{-\sigma}$, the probability that he meets exactly one buyer is $\sigma e^{-\sigma}$, and the probability that he meets at least two buyers is $1 - e^{-\sigma} - \sigma e^{-\sigma}$, where σ is the buyer-seller ratio. The probability that a buyer is the only buyer at the seller who she arrives at is $e^{-\sigma}$.

I assume, however, that buyers choices in the search market are determined by prices sellers post. I consider a symmetric strategy in pricing. Kultti (1999) shows that this is utilitywise equivalent to a model where buyers choose sellers randomly, and the terms of trade are determined by an auction where (i) The buyer gets all the surplus of the trade, and the seller gets his reservation value (which is zero in the static model), if the seller meets only one buyer. (ii) The seller gets all the surplus of the trade, and the buyer gets his reservation value, if the seller meets at least two buyers. By the equivalency result, the search market is of an urn-ball type where buyers choose sellers randomly. Using the equivalence result simplifies the analysis considerably.

Let $x \in [0, 1]$ be the fraction of sellers who choose the search market, and let $y \in [0, 1]$ be the fraction of buyers who choose the search market. The rest of the agents choose the cluster. The Poisson term (buyer-seller ratio) in the search market is $\sigma \equiv y\theta/x$, and in the cluster the buyer-seller ratio is $\gamma \equiv (1 - y)\,\theta/(1 - x)$. Let $p \in [0, 1]$ be the price posted in the cluster, and let $q \in [0, 1]$ be the price posted in the search market. I assume that sellers use a symmetric strategy in pricing in both markets, and that they are committed to the prices. Let U_c be a buyer's expected utility in the cluster, and let U_s be a buyer's expected utility in the search market. Let V_c and V_s be sellers' expected utilities, respectively.

The agents make their choices sequentially. First, sellers choose which market they go to, and they post a price. The sellers make an expectation whether they will end up on the short side or on the long side of the cluster, that is, either $E[\gamma] > 1$ or $E[\gamma] < 1$. This is equivalent to making an expectation of y, given the values of x and p. I assume that sellers make a rational expectation of y. Second, buyers observe sellers' locations and prices, and they go to the cluster or to the search market.

A mixed market equilibrium, where $x \in (0, 1)$ and $y \in (0, 1)$, exists only if buyers and sellers are indifferent between the markets, that is, if $U_c = U_s$ and $V_c = V_s$. If $E[y] < 1 - 1/\theta$ (which is possible only if $\theta < 1$), then $E[\gamma] > 1$: sellers are on the short side of the cluster even if all sellers were there. Then sellers in the cluster can post $p = 1$ and still trade with probability one. In equilibrium all sellers choose the cluster and post $p = 1$. The second necessary condition for the existence of a mixed market equilibrium exists is thus $E[y] > 1 - 1/\theta$.

In principle, three equilibria can emerge: All agents are in the cluster, all agents are in the search market, or the equilibrium is of a mixed market type such that there are buyers and sellers in both markets. I show that a mixed market equilibrium exists if the price in the cluster is within a specific interval determined by the value of θ. It will turn out that there are two mixed market equilibrium continua: One where the agents expect that in the cluster there are more buyers than sellers, and another where they expect that sellers will be the larger population in the cluster.

The key difference of the present model to that of Kultti (2011) is that the total buyer-seller ratio, θ, is deterministic and common knowledge. In Kultti's model θ is a random variable, and sellers choose their locations as well as prices in the cluster without knowing its value.

3 A Low-Price Mixed Market Equilibrium

I consider first a mixed market equilibrium where the number of sellers in the cluster is smaller than the number of buyers there, that is, sellers are on the short side of the cluster. Fraction x of sellers chooses the search market, and the rest of the sellers choose the cluster. Sellers' expectation of buyers' choices is $E[y]$. Assume that sellers expect to form the short side of the cluster, that is, $E[\gamma] = \frac{(1-E[y])\theta}{1-x} > 1$. Then $E[y] \in (1-1/\theta, (x+\theta-1)/\theta)$. I assume that sellers' expectation of y is correct: $E[y] = y$, and $E[\gamma] = \gamma = \frac{(1-y)\theta}{1-x} > 1$.

As I use the equivalence result, q is suppressed from the analysis. The value functions are

$$U_c = \frac{1-p}{\gamma} = \frac{(1-p)(1-x)}{(1-y)\theta}, \quad (1)$$

$$U_s = e^{-\frac{y\theta}{x}}, \quad (2)$$

$$V_c = p, \quad (3)$$

$$V_s = 1 - e^{-y\theta/x} - \frac{y\theta}{x}e^{-y\theta/x}. \quad (4)$$

In the cluster buyers trade with probability $1/\gamma$ and pay p. In the search market a buyer's expected utility is $U_s = e^{-y\theta/x}$. As sellers expect to be on the short side of the cluster, they expect to trade with probability one, and their expected utility is then p. In the search market a seller's profit is one if he meets at least two buyers, and the probability of this occasion is $1 - e^{-y\theta/x} - \frac{y\theta}{x}e^{-y\theta/x}$. If he meets at most one buyer, his profit is zero.

There are buyers in both markets only if they are indifferent between the markets, that is, if $U_c = U_s \Leftrightarrow 1 - (1-y)\theta e^{-y\theta/x}/(1-x) = p$. On a buyer's indifference curve $\frac{dy}{dx} = \frac{(1-y)(x^2+(1-x)y\theta)}{x(1-x)(x+(1-y)\theta)} > 0$. Likewise, there are sellers in both markets only if they are indifferent between them, that is, if $V_c = V_s$. On a seller's indifference curve $\frac{dy}{dx} = \frac{y}{x} > 0$. A mixed market equilibrium where $U_c = U_s$ and $V_c = V_s$ satisfies

$$y = \frac{x(x+\theta-1)}{\theta}, \quad (5)$$

$$p = 1 - (\theta+x)e^{1-\theta-x}. \quad (6)$$

Equation (5) determines the equilibrium locus: the equilibrium values of x and y for various values of p. If $\theta < 1$, then $y > 0$ iff $x > 1-\theta$. If $\theta > 1$, then $y > 1-1/\theta$ iff $x > \hat{x}$.

We have $\frac{dy}{dx} = \frac{1}{\theta}(2x + \theta - 1) > 0$ if $x > 1 - \theta$ or if $\theta > 1$. Also, $\frac{d(y/x)}{dx} > 0$, and $x - y = \frac{x(1-x)}{\theta} > 0$. Equation (6) determines the equilibrium value of x for a given p. By (6), $dx/dp = \left((x + \theta - 1) e^{1-\theta-x}\right)^{-1} > 0$. The higher the price in the cluster the more there are buyers and sellers in the search market, and the larger buyer-seller ratio $y\theta/x$ there. Using (5) in $\gamma = \frac{(1-y)\theta}{1-x}$ gives $\gamma = x + \theta$ which is larger than one if $y > 0$. The higher the price in the cluster the smaller the cluster, and the larger the buyer-seller ratio there.

A mixed market equilibrium exists only if $y > 1 - 1/\theta$. This holds if $\theta < 1$. If $\theta > 1$, then $y > 1 - 1/\theta$ if $x^2 + (\theta - 1)x + 1 - \theta > 0$, which holds if

$$x > \frac{1}{2}\left(1 - \theta + \sqrt{(\theta - 1)(\theta + 3)}\right) \equiv \hat{x}. \tag{7}$$

Equations (5) and (6) show that there is a unique (x, y) for a given p, and x and y increase in p. The following lemma gives an interval for p such that a mixed market equilibrium exists:

Lemma 1 *(i) If $\theta < 1$, then $p_{\min} = 0$ and $p_{\max} = 1 - e^{-\theta} - \theta e^{-\theta}$. If $p = p_{\min}$, then $x = 1 - \theta$ and $y = 0$. If $p = p_{\max}$, then $x = y = 1$. (ii) If $\theta \geq 1$, then $p_{\min} = 1 - (\theta + \hat{x}) e^{1-\theta-\hat{x}}$, and $p_{\max} = 1 - e^{-\theta} - \theta e^{-\theta}$. If $p = p_{\min}$, then $x = \hat{x}$, and $y = 1 - 1/\theta$. If $p = p_{\max}$, then $x = y = 1$.*

Proof. (i) Set $x = 1 - \theta$, then $y = 0$ and $p = 0$. Set $x = 1$, then $y = 1$ and $p = 1 - e^{-\theta} - \theta e^{-\theta}$. (ii) Set $x = \hat{x}$, then $y = 1 - 1/\theta$, and $p = 1 - (\theta + \hat{x}) e^{1-\theta-\hat{x}}$. Set $x = 1$, then $y = 1$ and $p = 1 - e^{-\theta} - \theta e^{-\theta}$. ■

If $\theta < 1$, the left endpoint of the equilibrium locus is $(1 - \theta, 0)$, and the right endpoint is $(1, 1)$. If $\theta > 1$, the left endpoint of the equilibrium locus is $(\hat{x}, 1 - 1/\theta)$, and the right endpoint is $(1, 1)$.

Denoting the values of x and y by x_l and y_l we collect these findings in

Proposition 1 *If $\theta < 1$ and $0 < p < 1 - e^{-\theta} - \theta e^{-\theta}$, or if $\theta \geq 1$ and $1 - (\theta + \hat{x}) e^{1-\theta-\hat{x}} < p < 1 - e^{-\theta} - \theta e^{-\theta}$, there exists a mixed market equilibrium where $x_l \geq 1 - \theta$, $y_l = x_l (x_l + \theta - 1)/\theta$, and $p = 1 - (\theta + x_l) e^{1-\theta-x_l}$. The higher the price in the cluster the smaller the cluster and the larger the search market. In equilibrium $x_l > y_l$ and $dy_l/dx_l > 0$.*

Figure 1 depicts a low-price mixed market equilibrium in a case where $\theta < 1$ and in a case where $\theta > 1$. In Appendix A1 I show that if $p > 1 - e^{-\theta} - \theta e^{-\theta}$ and sellers are indifferent between the markets, then buyers are better off in the cluster because the buyer-seller ratio in the search market is too large.

The following notion justifies calling this equilibrium a *low-price mixed market equilibrium:*

Remark 1 *The price in the cluster is lower than the price in the search market.*

Proof. Use the result on equivalence of auction and posted prices. In the search market a seller meets at least one buyer with probability $1-e^{-\sigma}$. If sellers post price q, a seller's expected value is $(1-e^{-\sigma})\,q$. In the auction setting $V_s = 1-e^{-\sigma}-\sigma e^{-\sigma}$. The utilities are the same, thus $q = \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$ where $\sigma = \dfrac{y\theta}{x} = x+\theta-1$, where the latter equation is given by (5). In the cluster $p = 1-(\theta+x)\,e^{1-\theta-x}$. Then $q-p = \dfrac{pe^{1-\theta-x}}{1-e^{1-\theta-x}} > 0$. ■

In a low-price equilibrium buyers outnumber sellers in the cluster, thus sellers in the cluster trade with probability one. One might then wonder whether sellers in the cluster would like to choose a higher price. At a higher p buyers in the cluster would fare worse than before, and the fraction of buyers who choose the search market would be larger than before. A higher p would increase V_c, but a larger number of buyers in the search market would make V_s larger as well. By (5) and (6), buyers and sellers would be indifferent between the markets at higher values of x and y, that is, the search market will be larger. As a result, sellers fare better than they did when they chose the lower p.

If only a search market is available, then $U = e^{-\theta}$ and $V = 1-e^{-\theta}-\theta e^{-\theta}$. In a mixed market equilibrium $U_c = U_s = e^{1-\theta-x} > e^{-\theta}$ and $V_s = V_c = p = 1-(\theta+x)\,e^{1-\theta-x} < 1-e^{-\theta}-\theta e^{-\theta}$. In a mixed market equilibrium where sellers are on the short side of the cluster, buyers fare better and sellers fare worse than in a pure search market.

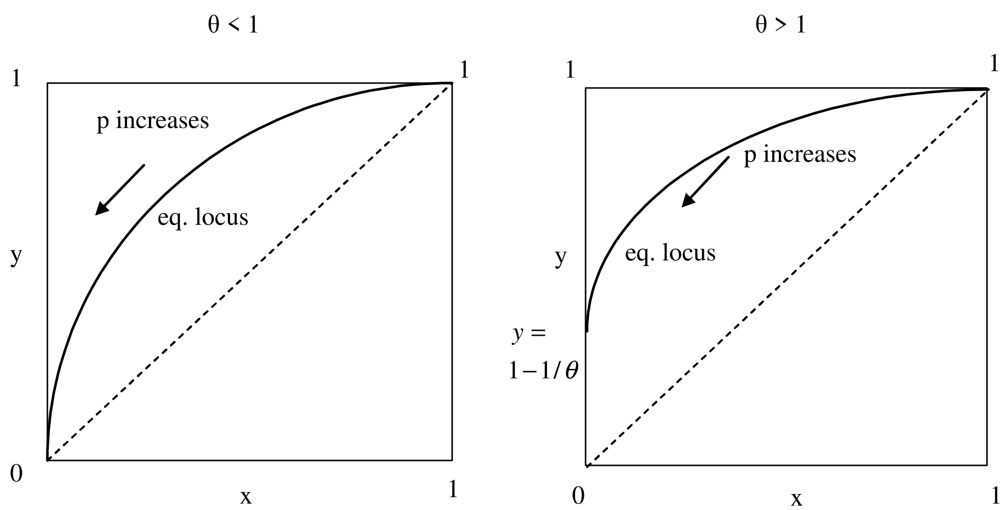

Figure 1: A low-price mixed market equilibrium

Before continuing to analyzing a high-price equilibrium, I make some notions:

(i) Equation (6) gives $dx/d\theta = -1$. The larger θ the closer to the diagonal (where $y = x$) the equilibrium locus will locate. (ii) The number of trades per seller is $m = 1-x_l+x_l\left(1-e^{-y_l\theta/x_l}\right)$. Using $y = x_l\,(x_l+\theta-1)/\theta$ gives $m = 1-x_le^{1-\theta-x_l}$ which decreases in x_l. The higher the price in the cluster the larger the search market and the less there are trades. (i) If $\theta < 1$, then $p \to 0$ gives $x \to 1-\theta$, and then $m \to \theta$. Also, $p \to 1-e^{-\theta}-\theta e^{-\theta}$ gives $x \to 1$, and then $m \to 1-e^{-\theta}$. (ii) If $\theta > 1$, then $x \to \hat{x}$ gives

$m \to 1 - \frac{1}{2}\left(1-\theta+\sqrt{(\theta-1)(\theta+3)}\right) e^{1-\theta-\frac{1}{2}\left(1-\theta+\sqrt{(\theta-1)(\theta+3)}\right)}$. If $p \to 1 - e^{-\theta} - \theta e^{-\theta}$ then $x \to 1$, and $m \to 1 - e^{-\theta}$.

4 A High-Price Mixed Market Equilibrium

Next I consider a mixed market equilibrium where sellers are on the *long* side of the cluster. It will turn out that the price in the cluster will be higher than the price in the search market. Also, in this equilibrium the price in the cluster will be higher than the price in the low-price equilibrium.

Sellers' expectation of γ is $E[\gamma] = \frac{(1-E[y])\theta}{1-x} < 1$. Then $E[y] \in ((x+\theta-1)/\theta, 1)$. I assume that sellers' expectation of y is correct: $E[y] = y$, and $E[\gamma] = \gamma = \frac{(1-y)\theta}{1-x} < 1$. The value functions are

$$U_c = 1-p, \tag{8}$$

$$U_s = e^{-\frac{y\theta}{x}}, \tag{9}$$

$$V_c = \gamma p = \frac{(1-y)\theta p}{1-x}, \tag{10}$$

$$V_s = 1 - e^{-y\theta/x} - \frac{y\theta}{x} e^{-y\theta/x}. \tag{11}$$

As buyers are on the short side of the cluster, they trade with probability one and pay p. The values of agents in the search market have the same expressions as in the low-price case. Then $U_c = U_s$ if $y = -(x/\theta)\ln(1-p)$, and on a buyer's indifference curve $dy/dx = -(1/\theta)\ln(1-p) > 0$. As sellers expect to be on the long side of the cluster they trade with probability γ and are paid p. On a seller's indifference curve $V_c = V_s$ we have $\frac{dy}{dx} = \frac{px^3(1-y) + (1-x)^2 y^2 \theta e^{-\frac{y\theta}{x}}}{x(1-x)\left(px^2 + (1-x) y\theta e^{-\frac{y\theta}{x}}\right)} > 0.$

In a mixed market equilibrium where $U_c = U_s$ and $V_c = V_s$ the values of x and y turn out to be

$$x = \frac{(1-\theta)p + (1-p)\ln(1-p)}{p + \ln(1-p)}, \tag{12}$$

$$y = \frac{(\theta-1)p\ln(1-p) - (1-p)\ln^2(1-p)}{\theta(p+\ln(1-p))}. \tag{13}$$

Equations (12) and (13) determine the equilibrium locus (x, y) for various values of p. I show in Appendix A2 that $dx/dp < 0$ and $dy/dp < 0$. There is thus a unique mixed market equilibrium for given p and θ. The higher the price in the cluster the smaller the search market and the larger the cluster.

Equations (12) and (13) give an interval for p such that a mixed market equilibrium exists:

Lemma 2 *(i) If $\theta > 1$, a mixed market equilibrium exists if $p \in \left(1 - e^{-\theta}, 1\right)$.*
(ii) If $\theta < 1$, a mixed market equilibrium exists if $p \in \left(1 - e^{-\theta},\, p_{\max}\right)$, where $(1/p_{\max})\left(p_{\max} + (1 - p_{\max}) \ln(1 - p_{\max})\right) = \theta$.

Proof. First, $p + \ln(1 - p) < 0 \;\forall\; p \in (0, 1)$. Then $x > 0$ and $y > 0$ if $(1/p)\left(p + (1 - p)\ln(1 - p)\right) < \theta$. This holds for all $\theta > 1$ because $(1/p)\left(p + (1 - p)\ln(1 - p)\right) < 1$. If $\theta < 1$ then $x > 0$ and $y > 0$ if $p < p_{\max}$ where $(1/p_{\max})\left(p_{\max} + (1 - p_{\max})\ln(1 - p_{\max})\right) = \theta$. If $x > 0$ and $y > 0$ then $x < 1$ and $y < 1$ if $p > 1 - e^{-\theta}$. ■

Equations (12) and (13), together with $p > 1 - e^{-\theta}$ give $y - x = -\left(\theta + \ln(1 - p)\right)(x/\theta) > 0$. In Appendix A3 I show that on a high-price mixed market equilibrium locus $dy/dx > 0$, and in Appendix 4 I show $d(y/x)/dx < 0$. The higher the price in the cluster the smaller the search market and the larger the value of Poisson term $y\theta/x$. That is, the higher p the tighter the search market is for buyers and the slacker it is for sellers. Using (12) and (13) gives $\gamma = (1/p)\left(p + (1 - p)\ln(1 - p)\right) < 1$, and $d\gamma/dp > 0$. The higher the price in the cluster the larger it is, and the the tighter the cluster is for buyers. Opposed to the low-price case, a higher price in the cluster increases the size of the cluster and decreases the size of the search market. As in the low-price case, a higher price in the cluster makes both markets tighter for buyers and slacker for sellers.

Equations (8) – (11) give the endpoints of the equilibrium locus:

Lemma 3 *If $\theta < 1$, the left endpoint of the equilibrium locus (x, y) is $(0, 0)$. If $\theta \geq 1$, the left endpoint of the equilibrium locus (x, y) is $(0, 1 - 1/\theta)$. The right endpoint of (x, y) is $(1, 1)$ for all $\theta > 0$.*

Proof. Equations (8) – (11) give $U_c = U_s$ and $V_c = V_s$ if $\left(1 - e^{-y\theta/x} - (y\theta/x)\, e^{-y\theta/x}\right)(1 - x) - (1 - y)\,\theta\left(1 - e^{-y\theta/x}\right) \equiv h = 0$. (i) Suppose $x \to 0$ and $y \to 0$. Then $h = \left(1 - e^{-y\theta/x} - (y\theta/x)\, e^{-y\theta/x}\right) - \theta\left(1 - e^{-y\theta/x}\right) \to 0$ only if $\theta < 1$.(ii) Suppose $x \to 0$ and $y > 0$. Then $e^{-y\theta/x} \to 0$ and $(y\theta/x)\, e^{-y\theta/x} \to 0$. Then $h = 1 - (1 - y)\,\theta \to 0$ which holds only if $y \to 1 - 1/\theta$, which is possible only if $\theta \geq 1$. (iii) The case $x > 0$ and $y \to 0$ is not possible since $y > x$. (iv) Suppose $x \to 1$. Then $h = (y - 1)\,\theta\left(1 - e^{-y\theta}\right) \to 0$ only if $y \to 1$ or if $y \to 0$, but $y \to 0$ violates $y > x$. Then $y \to 1$ if $x \to 1$. ■

Denoting the values of x and y by x_h and y_h we collect these findings in

Proposition 2 *(i) If $\theta < 1$, a mixed market equilibrium where $x_h \in (0, 1)$ and $y_h \in (0, 1)$ exists if $p \in \left(1 - e^{-\theta}, p_{\max}\right)$, where $(1/p_{\max})\left(p_{\max} + (1 - p_{\max})\ln(1 - p_{\max})\right) = \theta$. (ii) If $\theta \geq 1$, a mixed market equilibrium where $x_h \in (0, 1)$ and $y_h \in (1 - 1/\theta, 1)$ exists if $p \in \left(1 - e^{-\theta}, 1\right)$. (iii) On the equilibrium continuum $y_h > x_h$ and $dy_h/dx_h > 0$.*

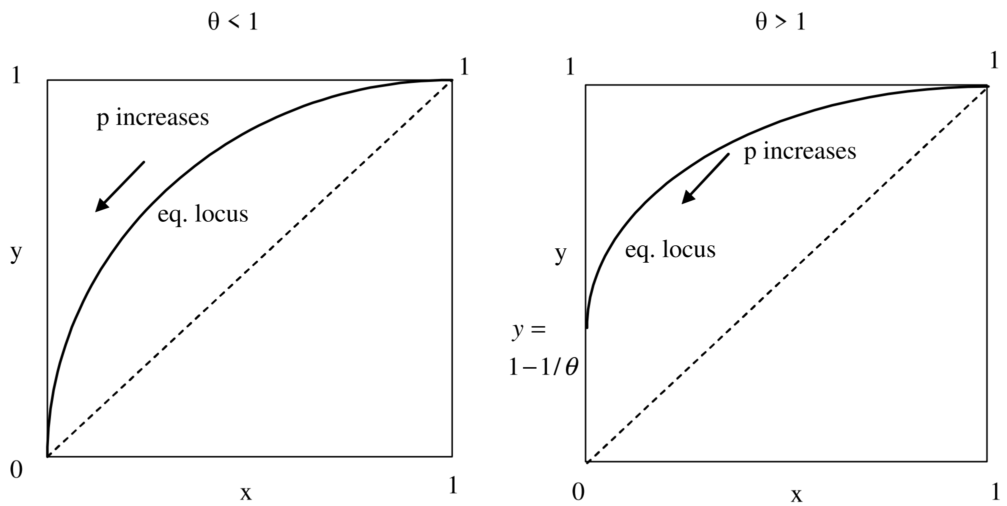

Figure 2: A high-price mixed market equilibrium

Notice that if $\theta > 1$,there is an equilibrium where all sellers are in the cluster, and the fraction of buyers who are in the cluster is larger than $1/\theta$. In this equilibrium $p = 1$, and buyers have $U_c = U_s = 0$.

In Appendix A5 I show that if $p < 1 - e^{-\theta}$ and buyers are indifferent between the markets, then sellers are better off in the cluster. If the price in the cluster is low, buyers' indifference condition gives a low value for y/x, which gives a low value for V_s. If $\theta < 1$, then $p > p_{\max}$, and buyers are indifferent between the markets, then sellers are better off in the search market. If the price in the cluster is high, buyers' indifference condition gives a large value for y/x, which gives a large value for V_s. By the following notion we name this equilibrium as *high-price mixed market equilibrium:*

Remark 2 *The price in the cluster is higher than the price in the search market.*

Proof. In the cluster $p = 1 - e^{-\sigma}$, and in the search market $q = \dfrac{1 - e^{-\sigma} - \sigma e^{-\sigma}}{1 - e^{-\sigma}}$. Then $p - q = \dfrac{e^{-\sigma}\left(\sigma + e^{-\sigma} - 1\right)}{1 - e^{-\sigma}} > 0$. ■

In the high-price equilibrium sellers outnumber buyers in the cluster. Now one might think that sellers in the cluster could benefit from posting a lower price in order to increase the probability of trading. If all sellers in the cluster chose a lower price, then buyers and sellers would be indifferent between the markets at larger values of x and y. However, V_s increases in y/x which, in turn, decreases in x. If sellers in the cluster attempt to benefit from posting a lower p, this results in a larger search market and lower expected utility for sellers.

A low-price equilibrium emerges if sellers expect to be on the short side of the cluster, and a high-price equilibrium emerges if sellers expect form the long side of the cluster. As the low-price equilibrium emerges only if $p < 1 - e^{-\theta} - \theta e^{-\theta}$ and the high-price equilibrium emerges only if $p > 1 - e^{-\theta}$, we have $p_h > p_{l.}$ for a given value of θ. Notice that a mixed market equilibrium does not exist if $1 - e^{-\theta} - \theta e^{-\theta} < p < 1 - e^{-\theta}$.

If only a search market existed, then $U = e^{-\theta}$ and $V = 1 - e^{-\theta} - \theta e^{-\theta}$. In a high-price mixed market equilibrium the expected utilities are $U_c = U_s = e^{-\sigma}$ and $V_c = V_s = 1 - e^{-\sigma} - \sigma e^{-\sigma}$, where $\sigma = y_h \theta / x_h$ where $y_h > x_h$.Then $\sigma > \theta$, and $U_c = U_s < e^{-\theta}$ and $V_c = V_s > 1 - e^{-\theta} - \theta e^{-\theta}$. In a mixed market equilibrium where sellers are on the long side of the cluster, buyers fare worse and sellers fare better than in a pure search market.

Notice also that (i) The equilibrium value of y_h decreases in θ, for a given x (see Appendix A6). The larger θ the further from the diagonal (where $y = x$) the equilibrium locus will be. (ii) The lower the price in the cluster the larger the search market and the less there are matches. In the cluster there are $S(1 - x_h)$ sellers who trade with probability $(1-y)\,\theta/(1-x)$. In the search market there are Sx_h sellers who trade with probability $1 - e^{-y_h \theta / x_h}$. Summing up the number of trades per seller in each market gives $m_h = (1 - y_h)\,\theta + x_h\left(1 - e^{-y_h\theta/x_h}\right)$. Using (12) and (13) gives $m_h = p + (1-p)\,(\theta + \ln(1-p))$. If $p_h \to 1 - e^{-\theta}$ then $m_h \to 1 - e^{-\theta}$. If $\theta > 1$ and $p_h \to 1$ then $m_h \to 1$. If $\theta < 1$ and $p_h \to p_{\max}$ then $m_h \to \theta$.

5 Stability Analysis

In this section I study the stability of a mixed market equilibrium. Although the model is static, a local stability analysis can be performed if we assume consecutive generations of one period living agents. All agents exit the economy after one period, and they are replaced by identical but unmatched agents. A fraction of the entrants chooses the market where their type fared best in the previous period. The rest choose the market of their predecessors. Myopic adjustment is natural in economies with a large number of agents where coordination is difficult. The resulting replicator dynamics is commonly used in evolutionary game theory (e.g. Weibull, 1995). Following Lu and McAfee (1996), we avoid the possibility of overshooting by assuming that changes in x and y are small (see Nachbar, 1990).

The replicator dynamics works as follows: Let the buyers' and sellers' average expected utilities be U and V, given their choices y and x: $U = yU_s + (1-y)U_c$, and $V = xV_s + (1-x)V_c$. The population shares can be assumed to adjust according to differential equations $dy/dt = y(U_s - U) = y(1-y)(U_s - U_c)$, and $dx/dt = x(V_s - V) = x(1-x)(V_s - V_c)$. The dynamics can be performed graphically using buyers' and sellers' indifference curves and the equilibrium locus where $U_c = U_s$ and $V_c = V_s$.

5.1 Stability Analysis of a Low-Price Equilibrium

I begin by showing that buyers' indifference curve is steeper than sellers' indifference curve, and that the equilibrium locus is steeper than buyers' indifference curve, all calculated in an intersection of the curves. On a buyer's indifference curve $\frac{dy}{dx} = \frac{(1-y)\left(x^2 + (1-x)\,y\theta\right)}{x\,(1-x)\,(x + (1-y)\,\theta)}$, and on a seller's indifference curve $\frac{dy}{dx} = \frac{y}{x}$. We have

$\dfrac{dy}{dx}(U_c = U_s) - \dfrac{dy}{dx}(V_c = V_s) = \dfrac{x-y}{(1-x)(x+(1-y)\theta)} > 0$ because $x > y$. That is, buyers' indifference curve is steeper than sellers' indifference curve.

On the equilibrium locus $y = (x/\theta)(x+\theta-1)$, which gives $\dfrac{dy}{dx}(U_C = U_S$ and $V_C = V_S) = (2x+\theta-1)/\theta$. Calculate next $\dfrac{dy}{dx}(U_C = U_S$ and $V_C = V_S) - \dfrac{dy}{dx}(U_c = U_s)$. In the latter term we use $y = (x/\theta)(x+\theta-1)$, and the difference is $\dfrac{x(1-x)(x+\theta-1)}{\theta(\theta(1-x)+2x-x^2)}$ where $x+\theta-1 > 0$ whenever $y > 0$. Then $\theta(1-x)+2x-x^2 > (1-x)^2+2x-x^2 = 1$. That is, the equilibrium locus is steeper than a buyer's indifference curve.

Figure 3 depicts the analysis. On the right of buyers' indifference curve x is "too" large, that is, there are "too many" sellers in the search market. By (1) and (2) we have $U_s > U_c$, and the fraction of buyers who choose the search market grows over time, that is, y increases. The opposite holds on the left of buyers' indifference curve. Above sellers' indifference curve y is "too" large (there are "too many" buyers in the search market). By (3) and (4) $V_s > V_c$, and then the fraction x of sellers who choose the search market grows.

The equilibrium is in E_0 if sellers have chosen p_0 in the cluster. Consider a situation that for some reason, the initial values of x and y do not correspond to an equilibrium. If x and y start outside of E_0, the arrows depict the changes in x and y as consecutive generations of buyers and sellers choose their markets. The outcome resembles of a saddlepoint equilibrium. However, sellers choose also the price p in the cluster. This makes it possible to reach an equilibrium from outside a saddle path leading to E_0. The key is that the equilibrium locus does not fall in between the indifference curves. Suppose x and y start at a point between the upper saddle path and upper part of the equilibrium locus, for example at point A_1 where $x = x_0$ but $y > y_0$. Then $U_c > U_s$ and $V_c < V_s$. According to the replicatory dynamics, x increases and y decreases. Eventually x and y hit the equilibrium locus at E_1. Point E_1 is an intersection of new indifference curves at price $p_1 > p_0$ where (x_1, y_1, p_1) satisfy $U_c = U_s$ and $V_c = V_s$. At the very moment x and y hit the equilibrium locus, sellers could choose p_1, and the adjustment is ceased. Also, if x and y start between the lower saddle path and the lower part of the equilibrium locus, for example at A_2 where $x = x_0$ but $y < y_0$, a new equilibrium E_2 will be reached if sellers choose $p_2 < p_0$, where (x_2, y_2, p_2) satisfy $U_c = U_s$ and $V_c = V_s$, at the moment x and y hit the equilibrium locus.

If the initial (x, y) is not between the upper saddle path and the upper part of the equilibrium locus nor between the lower saddle path and the lower part of equilibrium locus, a new mixed market equilibrium cannot be reached with any price adjustment. For example, if the economy starts at A_3 where $y = y_0$ but $x > x_0$, then x will first decrease and y will increase. After the adjustment path has crossed sellers' indifference curve, x and y will increase. Starting at A_4 where $y = y_0$ but $x < x_0$, x will first increase and y will decrease, but after the adjustment path has crossed sellers' indifference curve, both x and y decrease.

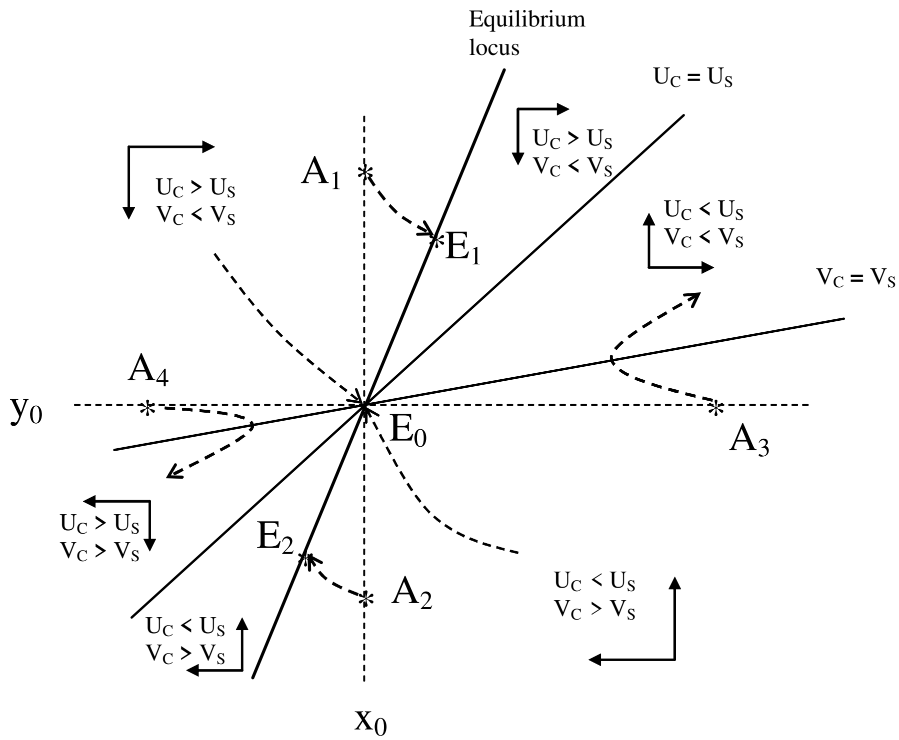

Figure 3: The stability analysis of a mixed market equilibrium

5.2 Stability Analysis of a High-Price Equilibrium

I show that the equilibrium locus is steeper than a buyer's indifference curve, which is again steeper than a seller's indifference curve. On buyers' indifference curve $\frac{dy}{dx}(U_c = U_s)$

$= -\frac{\ln(1-p)}{\theta} = \frac{y}{x}$. The last equation results from using (12) and (13). The value of dy/dx on a sellers' indifference curve is solved in Section 2. Then $\frac{dy}{dx}(U_c = U_s) - \frac{dy}{dx}(V_c = V_s) = \frac{px(y-x)}{(1-x)\left(px^2 + y\theta(1-x)e^{-y\theta/x}\right)} > 0$ because $y > x$. That is, buyers' indifference curve is steeper than sellers' indifference curve in their intersection.

On the equilibrium locus where $U_c = U_s$ and $V_c = V_s$ we have
$1 - e^{-y\theta/x} - \frac{1}{\theta}\frac{1-x}{1-y}\left(1 - e^{-y\theta/x} - \frac{y\theta}{x}e^{-y\theta/x}\right) = 0$. This gives

$$\frac{dy}{dx}(U_c = U_s, V_c = V_s) = \frac{(1-y)\left(\left(x^3 - y^2\theta^2 + xy\theta^2 + x^2y\theta\right)e^{-y\theta/x} - x^3\right)}{\left(x(1-x)(x+y\theta) - (1-y)(y-x)\theta^2\right)xe^{-y\theta/x} - (1-x)x^3}$$

which is positive as already shown. Then

$$\frac{\frac{dy}{dx}\left(U_c = U_s, V_c = V_s\right)}{\frac{dy}{dx}\left(U_c = U_s\right)} = \frac{x\left(1-y\right)\left(\left(x^3 - y^2\theta^2 + xy\theta^2 + x^2 y\theta\right)e^{-y\theta/x} - x^3\right)}{\left(x\left(1-x\right)\left(x+y\theta\right) - \left(1-y\right)\left(y-x\right)\theta^2\right)xye^{-y\theta/x} - \left(1-x\right)x^3 y}.$$

The numerator minus the denominator equals $x^3\left(y-x\right)\left(1 - e^{-y\theta/x} - \frac{y\theta}{x}e^{-y\theta/x}\right) > 0$ because $y > x$ in a high-price equilibrium. Just like in the low-price case, the equilibrium locus is steeper than buyers' indifference curve which, in turn, is steeper than sellers' indifference curve. Also here, on the right of buyers' indifference curve x is "too" large, and y will increase as new generations of buyers tend to choose the search market more probably than their predecessors. The opposite holds on the left of buyers' indifference curve. Above sellers' indifference curve y is "too" large, thus the fraction sellers who choose the search market grows. The stability properties of a high-price equilibrium are the same as those of a low-price equilibrium. If x and y start between the upper saddle path and upper part of the equilibrium locus, x increases and y decreases until they hit the equilibrium locus at E_1. Point E_1 is an intersection of new indifference curves at price p_1, but now $p_1 < p_0$. Choices (x_1, y_1, p_1) satisfy $U_c = U_s$ and $V_c = V_s$. At the very moment x and y hit the equilibrium locus, sellers could choose p_1, and the adjustment is ceased. If x and y start between the lower saddle path and the lower part of the equilibrium locus, for example at A_2 where $x = x_0$ but $y < y_0$, a new equilibrium E_2 will be reached if sellers choose $p_2 > p_0$, where (x_2, y_2, p_2) satisfy $U_c = U_s$ and $V_c = V_s$, at the moment x and y hit the equilibrium locus. As in the high-price case, if the initial (x, y) is not between the upper saddle path and the upper part of the equilibrium locus nor between the lower saddle path and the lower part of equilibrium locus, a new mixed market equilibrium cannot be reached with any price adjustment.

6 Pure Strategy Equilibria

Let us solve whether all agents in the cluster or in the search market is an equilibrium. Instead of standard Nash equilibrium test where a deviation of a single agent to the other market is considered, we must consider a coalitional deviation. This is because trading takes place between buyers and sellers, thus a single agent cannot benefit from switching to the other market.

Proposition 3 *All agents in the cluster is an equilibrium, but all agents in the search market is not an equilibrium.*

Proof. (i) Assume all agents are in the cluster. If $\theta > 1$, buyers compete for the goods, and all sellers can charge $p = 1$ and still trade. Then $V_c = 1$ and $U_c = 0$. If $\theta < 1$, sellers compete for the buyers, the price is driven to zero, and $V_c = 0$ and $U_c = 1$. In either case it is not possible that a coalition of buyers and sellers could choose the search market such that all members of the coalition are better off. If $\theta = 1$, all agents in the cluster trade. Then $V_c = p$ and $U_c = 1 - p$. Suppose that a coalition of η buyers and μ sellers deviates to the search market. Denote $\sigma = \eta/\mu$. A deviating buyer's expected utility is $U_d = e^{-\sigma}$, and a deviating seller's expected utility is $V_d = 1 - e^{-\sigma} - \sigma e^{-\sigma}$. Then $U_d > U_c$ if $p > 1 - e^{-\sigma}$, and $V_d > V_c$ if $1 - e^{-\sigma} - \sigma e^{-\sigma} > p$. Both equations cannot hold, and we conclude that all agents being in the cluster is an equilibrium.

(ii) Assume then that all agents are in the search market. The expected values of buyers and sellers are $U_s = e^{-\theta}$ and $V_s = 1 - e^{-\theta} - \theta e^{-\theta}$, respectively. Let fraction η of buyers and fraction μ of sellers form a coalition which deviates to the cluster where sellers set price p. Denote $\lambda \equiv \eta\theta/\mu$. Suppose $\lambda > 1$. Then in the cluster a seller trades with probability one, and a buyer trades with probability $1/\lambda$. The expected values of deviating agents are $V_d = p$ and $U_d = (1-p)/\lambda$. Then $U_d > U_s$ and $V_d > V_s$ if $1 - e^{-\theta} - \theta e^{-\theta} < p < 1 - \lambda e^{-\theta}$. This can hold only if $\eta/\mu \leq 1 + 1/\theta$. On the other hand, $\lambda > 1$ iff $\eta/\mu > 1/\theta$. A profitable deviation exists only if $1/\theta < \eta/\mu \leq 1 + 1/\theta$. The values of η and μ can be chosen such that this holds, and sellers in the cluster can choose p such that $1 - e^{-\theta} - \theta e^{-\theta} < p < 1 - \lambda e^{-\theta}$ holds. A deviating coalition thus exists such that all members of the coalition fare better than in the search market. That is, all agents being in the search market is not an equilibrium. ■

7 Conclusion

I have shown that a cluster and a search market can coexist in a rational expectations model where the numbers of buyers and sellers are deterministic. Sellers make an expectation of buyers' choices between the markets, and they choose markets and post binding prices. After that buyers choose which market to go to. The type of mixed market equilibrium depends on whether sellers expect to be on the short side or on the long side of the cluster. In the former case the mixed market equilibrium is of low-price type where the price in the cluster is lower than in the search market. If sellers expect to be on the long side of the cluster, the mixed market equilibrium is of high-price type where the price in the cluster is higher than in the search market. I also showed that in the high-price equilibrium the price in the cluster is higher than in the low-price equilibrium, for a given buyer-seller ratio. I solved intervals, determined by the total buyer-seller ratio, for the price in the cluster so that a mixed market equilibrium exists. The model exhibits also a pure strategy equilibrium where all agents are located in the cluster. A pure strategy equilibrium where all agents are in the search market does not exist. The reason is that a coalition of buyers and sellers can deviate to the cluster, and sellers can choose a price such that the deviators are better off in the cluster. The model is static, but I analyzed its stability properties by using replicator dynamics, where agents live one period, and then they are replaced by identical but unmatched agents. Both mixed market equilibria have the same stability properties. The equilibria are of saddle-point type, but if an adjustment path starts in a specific region, a new mixed market equilibrium can be reached if sellers adjust the price correctly.

8 Appendix

A1 In the low-price case we have $U_c - U_s = \dfrac{\left(x\left(1-x\right)+\left(y-x\right)\theta\right)e^{-y\theta/x}}{x\left(1-y\right)\theta}$ if $V_c = V_s$.

Let $V_c = V_s$ and $p \geq 1 - e^{-\theta} - \theta e^{-\theta}$. Then $1 - e^{-y\theta/x} - \dfrac{y\theta}{x}e^{-y\theta/x} \geq 1 - e^{-\theta} - \theta e^{-\theta}$, which gives $y \geq x$. Then $U_c > U_s$ at $x \in (0,1)$. That is, buyers are better off in the cluster. If p is large and $V_c = V_s$, then there must be relatively many buyers in the search market, and therefore $U_c > U_s$.

A2 In the high-price case $\dfrac{dx}{dp} < 0$ and $\dfrac{dy}{dp} < 0$:

(i) Equation (12) gives $\dfrac{dx}{dp} = \dfrac{p\left(\theta-p\right)+\left(1-p\right)\left(\theta+\ln\left(1-p\right)\right)\ln\left(1-p\right)}{\left(p-1\right)\left(p+\ln\left(1-p\right)\right)^2}$ where the denominator is negative. The numerator is positive if $\theta > \dfrac{p^2-\left(1-p\right)\ln^2\left(1-p\right)}{p+\left(1-p\right)\ln\left(1-p\right)}$. Also, $\theta > \dfrac{p+\left(1-p\right)\ln\left(1-p\right)}{p}$ because $x > 0$. Because $\dfrac{p+\left(1-p\right)\ln\left(1-p\right)}{p}$
$> \dfrac{p^2-\left(1-p\right)\ln^2\left(1-p\right)}{p+\left(1-p\right)\ln\left(1-p\right)}$ $\forall\, p \in (0,1)$, then
$\theta > \dfrac{p^2-\left(1-p\right)\ln^2\left(1-p\right)}{p+\left(1-p\right)\ln\left(1-p\right)}$, and then $\dfrac{dx}{dp} < 0$.

(ii) Equation (13) gives

$$\frac{dy}{dp} = \frac{\begin{array}{c}\left(p-1\right)\left(\ln^2\left(1-p\right)+\ln\left(1-p\right)+2p\right)\ln\left(1-p\right)-p^2\\ +\theta\left(p^2-\left(1-p\right)\ln^2\left(1-p\right)\right)\end{array}}{\theta\left(p-1\right)\left(p+\ln\left(1-p\right)\right)^2}$$

where the denominator is negative. Also, $p^2 - \left(1-p\right)\ln^2\left(1-p\right) > 0$, and
$\theta > \dfrac{p+\left(1-p\right)\ln\left(1-p\right)}{p}$ because $x > 0$. Then the numerator is larger than
$\dfrac{p-1}{p}\left(p+\ln\left(1-p\right)\right)^2\ln\left(1-p\right)$ which is positive, and then $\dfrac{dy}{dp} < 0$.

A3 I show that in the high-price mixed equilibrium $dy/dx > 0$.
On the equilibrium locus where $U_c = U_s$ and $V_c = V_s$ we have $h \equiv \left(1-y\right)\theta\left(1-e^{-y\theta/x}\right)$
$-\left(1-x\right)\left(1-e^{-y\theta/x}-\left(y\theta/x\right)e^{-y\theta/x}\right) = 0$, which gives
$\dfrac{dy}{dx} = \dfrac{x^3\left(1-e^{-y\theta/x}\right)-y\theta x^2 e^{-y\theta/x}+y\theta^2\left(y-x\right)e^{-y\theta/x}}{x^3\theta\left(1-e^{-y\theta/x}\right)+x\theta^2\left(y-x\right)e^{-y\theta/x}}$. Using $\sigma = y\theta/x$ and rearranging gives $\dfrac{dy}{dx} = \dfrac{x^2\left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)+\theta\left(y-x\right)\sigma e^{-\sigma}}{\theta x^2\left(1-e^{-\sigma}\right)+\theta^2\left(y-x\right)e^{-\sigma}} > 0$ because $y > x$.

A4 I show that $d\left(y/x\right)/dx < 0$ in a high-price equilibrium. Denoting $z = y/x$ equation $h = 0$ can be written as $g = \left(\theta+x-1\right)\left(1-e^{-z\theta}\right) - z\theta\left(x-e^{-z\theta}\right) = 0$. Then $\dfrac{dz}{dx} = -\dfrac{1-z\theta-e^{-z\theta}}{\left(1-z\right)\theta^2 e^{-z\theta}-x\theta\left(1-e^{-z\theta}\right)} < 0$ because $z > 1$ and $1 - z\theta - e^{-z\theta} < 0$.

A5 In the high-price case we have $V_c - V_s = \frac{(1-y)\,\theta p}{1-x} - \left(1 - e^{-y\theta/x} - (y\theta/x)\,e^{-y\theta/x}\right)$.
If $U_c = U_s$, then $y = -\frac{x \ln(1-p)}{\theta}$, and then $V_c - V_s = \frac{(x\ln(1-p)+\theta)\,p}{1-x}$
$-(p + (1-p)\ln(1-p))$.

(i) If $p < 1 - e^{-\theta}$, then $\theta > -\ln(1-p)$, and then $V_c - V_s > \frac{(x\ln(1-p) - \ln(1-p))\,p}{1-x}$
$-(p + (1-p)\ln(1-p)) = -p - \ln(1-p) > 0$.

(ii) If $p > \bar{p}$, then $\frac{p + (1-p)\ln(1-p)}{p} > \theta$, and then

$$V_c - V_s < \frac{\left(x\ln(1-p) + \frac{p+(1-p)\ln(1-p)}{p}\right)p}{1-x} - (p + (1-p)\ln(1-p))$$
$$= \frac{(p + \ln(1-p))\,x}{1-x} < 0.$$

A6 I show that $\frac{dy}{d\theta} > 0$ in a high-price equilibrium. Differentiating
$h \equiv (1-y)\,\theta\left(1 - e^{-y\theta/x}\right) - (1-x)\left(1 - e^{-y\theta/x} - (y\theta/x)\,e^{-y\theta/x}\right) = 0$ with respect to y and θ and then denoting $y\theta/x = \sigma$ gives $\frac{dy}{d\theta} = \frac{x^2(1-y)(1-e^{-\sigma}) - (y-x)\,x\sigma e^{-\sigma}}{\theta x^2(1-e^{-\sigma}) + \theta^2 (y-x)\,e^{-\sigma}}$
where the denominator is positive because $y > x$. Equation $h = 0$ can be written as $y = \frac{x\sigma(1-e^{-\sigma})}{1 - e^{-\sigma} - \sigma e^{-\sigma} + x(\sigma + e^{-\sigma} - 1)}$, and then the numerator is
$\frac{x^2(1-x)(1 - \sigma^2 e^{-\sigma} - 2e^{-\sigma} + e^{-2\sigma})}{1 - e^{-\sigma} - \sigma e^{-\sigma} - x(1 - \sigma - e^{-\sigma})} > 0$. Then $\frac{dy}{d\theta} > 0$.

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