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**Search in Networks: The Case of
Board Interlocks**

Aboa Centre for Economics

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ABSTRACT

We present a model for the dynamics of networks in which edges represent positions in organizations, holders of which are connected to each other when the positions belong to the same organization. Once a vacancy is opened, the new employee can be hired from the current network. In particular, it is possible that the further away a candidate is in the network from the firm having the vacant position, the less likely it is that the candidate is chosen. The search may also involve preferential attachment in the sense that people with high numbers of positions are more likely to be chosen. Microeconomic foundations of the search process are presented. An empirical application to a board interlock network demonstrates that the model is capable of explaining how such networks are formed. It is observed that distances from firms to candidates and the candidates' numbers of positions drive the process.

JEL Classification: D83; D85; Z13

JEL Classification: Keywords: search, two-mode network, network formation, board interlock network

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1. Introduction

The main challenge in analyzing the role of social networks in labour markets is the endogeneity arising from the present network affecting the future connections between agents. For example, a candidate who is found in the social network through a small number of intermediary persons, e.g., through referrals, may get hired, which in turn creates new ties between people and affects their prospects of getting new jobs in the future. Hence, the formation of networks is the key to understanding the properties of observed networks.

In this work the attention is on modeling the dynamics of two-mode networks in which there is a fixed number of positions, holders of which are connected to each other when the positions belong to the same organization. In principle, there are two sets of nodes representing different social entities: the individuals and the organizations. For example, in a board interlock network, which is our main application, one set of nodes is composed of board members, and the other of firms. These kinds of networks are known as two-mode networks or affiliation networks. The networks analyzed in this work are assumed to have more structure than just the division of agents into groups and individuals. The additional element is the set of positions or posts that is assumed to remain the same in time. In essence, each group has a given number of positions that can be occupied by the individuals of the network.

There is a variety of models for network formation and evolution, for an overview see Jackson (2005) and Newman (2003). Our approach relies on a search and matching type process, where once a vacancy is opened, the new holder of the position is searched from the current network or the vacancy is filled by choosing a person outside of the holders of current positions in the network. Hence, the search process presented in this work resembles the models of Jackson and Rogers (2007) and Bramoullé et al. (2012) that involve search of new connections in the neighbourhoods of randomly met nodes. In particular, the search is affected by the distance of people in the network from the firm having the vacancy and the number of positions that people have. The distance effect can result from a referral type of exchange of information. The number of positions, on the other hand, may signal the productivity of a candidate.

The search and matching in labor market networks has been widely studied in economics literature, the topic is surveyed by Ionnides and Loury (2004) and Topa (2011). For classical contributions on wage determination in search markets see Diamond (1981), Montgomery (1991, 1992), and Mortensen and Vishwanath (1994). There is also substantial literature on referrals as a

method for finding workers, see Dustmann et al. (2015) and Galenianos (2013) for recent works.

The search process presented in this paper has a number of distinguishing features that make it different from network search models presented in the previous literature. First, the focus is on the evolution of the network, while in many of the labor market search models the actual network structure plays no role or it is taken as exogenously given (Calvó-Armengol and Zenou, 2005; Fontaine, 2008), see, however, Boorman (1975), Calvó-Armengol (2004), and Galeotti and Merlini (2014) for notable exceptions. Second, in our model only the firms search for employees. Third, a crucial element of a two-mode network is that an individual may hold several positions in different organizations at the same time. Page and Wooders (2007, 2010) have studied the formation of this kind of networks in which the other mode represents clubs and the individuals choose strategically their club memberships. The fourth feature in our approach is that the set of individuals is not fixed; only the firms and the positions are assumed to stay the same.

Our model is in principle applicable to any two-mode network where the available positions are fixed. Such situation is typical for representative bodies of various organizations such as boards of companies or associations, editorial posts of journals, or various bodies of parliamentary systems (Porter et al., 2005). Collaboration networks, in which each member of a team has a particular role (Uzzi and Spiro, 2005), can be treated as two-mode networks with fixed numbers of available positions. In this work the attention is on board interlock networks, which have raised a lot of attention in social sciences beginning from the works of Galaskiewicz and Wasserman (1981) and Mintz and Schwartz (1981).

From an economics point of view board interlock networks or interlocking directorates are interesting for several reasons. Social networks affect market outcomes (Granowetter, 1973; Rees, 1966)—in particular, as argued by Saloner (1985), networks improve the quality of the director-management match, and hence corporate performance. On the other hand, social networks may have a detrimental effect to corporate governance (Hallock, 1997; Fich and Shivdasani, 2006; Kramarz and Thesmar, 2013), while at the same time there is evidence that companies with highly networked boards may perform better (Larcker et al., 2013).

This paper is structured as follows. Section 2 introduces the two-mode networks with given set of positions and the basic concepts used in the paper. The model for the evolution of the network is presented in Section 3, and its microeconomic foundations are studied in Section 4. Fitting the model to data is discussed in Section 5. Section 6 presents an empirical application which demonstrates that the model is able to capture the main features of a

real world network and matches the observations that new board members are found relatively close to the current board in the interlock network and they tend to hold multiple positions. Conclusions are discussed in Section 7.

2. Two-Mode Networks with Firms and Individuals

In a two-mode network there are two kinds of actors. In this paper these actors are assumed to be firms and individuals. There is a finite set of firms F with given positions V . Each position $v \in V$ is assumed to be held by some individual. The set of individuals is denoted by I . Each position belongs to some firm, let f_v stand for the firm having the position $v \in V$. The positions belonging to the firm $f \in F$ are $VF(f)$, and the positions held by the individual $i \in I$ are $VI(i)$. Individuals holding positions in firm f are denoted by the set $I(f)$.

Two positions are linked to each other if the same person holds them. Such persons are called interlockers. In particular, each $v \in V$ corresponds to a list of other positions held by the holder of v . Let V_v denote this list. In the following $n_v = |V_v|$ is the total number of positions held by the person who occupies the position v . It is assumed that if a position in a firm is held by some individual, all the other positions of that firm are occupied by some other people.

As an example, consider a network of nine positions labelled with v^1, \dots, v^9 and the set of firms $F = \{a, b, c\}$. The positions v^1, \dots, v^3 belong to firm a , v^4 and v^5 belong to firm b , and the rest belong to c . Hence, $f_{v^j} = a$ for $j = 1, \dots, 3$, $f_{v^j} = b$ for $j = 4, \dots, 5$, and $f_{v^j} = c$ for $j = 6, \dots, 9$. Moreover, assume that there are seven individuals $I = \{1, \dots, 7\}$ such that the first three have a position in firm a , the last four have a position in firm c , and individuals 1 and 4 hold also positions in firm b . The two-mode network corresponding to this example is illustrated in Figure 1. Note in particular that individuals 1 and 4 are interlockers with $VI(1) = \{v_1, v_4\}$ and $VI(4) = \{v_5, v_6\}$ or $V_{v^1} = V_{v^4} = \{v^1, v^4\}$ and $V_{v^5} = V_{v^6} = \{v^5, v^6\}$.

Note that in essence each position corresponds to an edge between a firm and an individual. What is crucial is that the number of these edges is limited, because each firm has a limited number of positions.

The holders of positions form a network; people having positions in the same firm are connected to each other, and two firms sharing a common individual are connected to each other. To describe the network composed of positions and their holders let us first consider the neighbourhoods of available positions in the network.

The immediate neighbours of $v \in V$ consist of all positions in the firm f_v , i.e., it is the set $N_0(v) = VF(f_v)$. The one-step neighbourhood of v contains

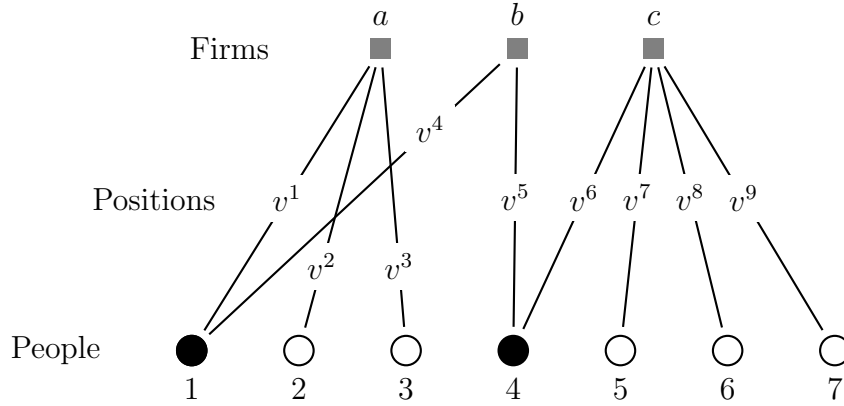


Figure 1: Example of a two-mode network with positions as edges. The interlockers are indicated with black circles.

all the positions in the firms which have board interlocks with the firm;

$$N_1(v) = \cup\{VI(i) : i \in I(f_v)\}.$$

Higher order neighbourhoods, the k -step neighbourhoods for $k \geq 2$, are obtained the same way;

$$N_k(v) = \cup\{VI(i) : i \in I(f_w), w \in N_{k-1}(v)\}.$$

In other words, $w \in N_k(v)$ corresponds to at most k interlockers through which v is connected to w .

To clarify the concept of a k -step neighbourhood let us continue the previous example. Regardless of the holders of the positions, the zero-step neighbourhoods are $N_0(v^j) = \{v^1, v^2, v^3\}$ for $j = 1, 2, 3$, $N_0(v^j) = \{v^4, v^5\}$ for $j = 4, 5$, and $N_0(v^j) = \{v^6, \dots, v^9\}$ for $j = 6, \dots, 9$. The k -steps neighbourhoods are

$$\begin{aligned} N_1(v^j) &= \{v^1, \dots, v^5\} \text{ for } j = 1, 2, 3, \\ N_1(v^j) &= V \text{ for } j = 4, 5, \\ N_1(v^j) &= \{v^4, \dots, v^9\} \text{ for } j = 6, \dots, 9, \end{aligned}$$

and $N_2(v^j) = V$ for all $j = 1, \dots, 9$.

Let $N(v)$ denote the list of all non-empty neighbourhoods of $v \in V$. To be specific, $N(v) = (N_0(v), N_1(v), \dots, N_{n-1}(v))$. A neighbourhood configuration at time t is the ordered list $\{N(v)\}_{v \in V}$. Because there are finitely many positions, there are also finitely many neighbourhood configurations. The set

of all possible neighbourhood configurations is denoted by \mathcal{N} . The projection of $N \in \mathcal{N}$ to the neighbourhoods of v is $N(v)$ with the corresponding space \mathcal{N}_v .

If w is found in some k -step neighbourhood of v but is not found in any j -step neighborhood for $j < k$, then the distance between v and w is $d_{w,v} = k$. If v is not found from any k -step neighbourhood of w , then $d_{w,v}$ is set to d^∞ . In the previous example $d_{v^1,v^j} = 1$ for $j = 4, 5$, and $d_{v^1,v^j} = 2$ for $j \geq 6$.

3. Evolution of Two-Mode Networks through the Filling of Vacancies

This section presents a model for the dynamics of two-mode networks with a given set of positions that are opened and filled as time goes on. As described in the previous section, the network is essentially determined by its neighbourhood configuration at each time instant. The purpose is to present simple probabilistic rules updating the neighbourhood configuration. It is assumed that time is discrete and it is indexed with $t = 0, 1, 2, \dots$. All variables that depend on the network, such as neighbourhoods of positions, are indexed with t , whenever needed.

The model for the evolution of two-mode networks is essentially a stochastic process for the edges and vertices of the network. The edges correspond to firm-individual pairs in which the individual holds a vacancy in the firm, and the vertices correspond to firms and individuals. Most of the existing models for network formation involve only dynamics of edges, while the set of possible vertices is kept fixed. Alternatively, it is often assumed that the network is growing as in the preferential-attachment model of Barabási and Albert (1999).

In our model there is no limited pool of people holding the positions. When time goes on arbitrarily many people may have held a vacancy. However, the network is not growing because the number of positions is kept fixed. In essence, opening and filling vacancies correspond to rewiring of edges of a two-mode network. See Evans and Plato (2007) for edge rewiring with a fixed set of agents, and Lafond (2015) for a model where the number of vertices in one of the modes is fixed while in the other mode the vertices are taken from a finite population.

3.1. Opening and filling of vacancies

It is assumed that vacancies open at random, the probability for position $v \in V$ to open in period t is $P_o(N^t(v))$. Note that the dependency on $N^t(v)$ means that the probability of opening the vacancy may depend on the number of positions that the previous holder of the position has had or how many

people the holder is connected to. If none of the positions held by the holder of v is opened then the list of positions held by this individual remains the same; $V_w^{t+1} = V_w^t$ for all $w \in V_v^t$.

When a vacancy opens, its holder is changed to a new person taken either outside of the current holders of available positions in the network or from the set of holders of other positions. The first case leads to $V_v^{t+1} = v$, i.e., the only position held by the new holder is v . Hence, the list of positions held by the holder of position $w \in V_v^t$ is updated such that v is removed from V_w^t . In the second case, when the new person for the vacancy v is chosen among the holders of other positions, there is some position $w \notin V_v^t$ holder of which gets the position v . In this case V_w^{t+1} is appended with v .

Recall that each position corresponds to an edge of the two-mode network. Hence, opening and filling the vacancies can be interpreted as an edge rewiring process, where the rewiring probabilities are conditional on the properties of edges, and the number and labels of nodes in the other mode of the two-mode network are not fixed.

Let us now make assumptions on the probabilities of opening and filling vacancies:

- (A1) $v \in V$ opens with probability $P_o(N(v))$ such that $P_o(N(v)) > 0$ for any $N(v) \in \mathcal{N}_v$.
- (A2) the vacancy is filled outside of the holders of other positions with probability $P_n > 0$,
- (A3) if v is filled from the set of holders of other positions, then the probability of choosing the holder of $w \in V \setminus V_v$, is $P_h(N(w), N(v))$ such that $P_h(N(w), N(v)) > 0$ if $d_{w,v} \geq 1$, and $P_h(N(w), N(v)) = 0$ if $d_{w,v} = 0$.

The last assumption means that there is a search and matching process which determines the new holder of a position. The holder of w is matched with the firm f_v with a probability that depends on positions of the holder of w in the network and how these positions are related to the neighbourhoods of the vacancy to be filled. Note that having $P_h(N(w), N(v)) = 0$ if $d_{w,v} = 0$ means that v cannot be filled by any person already affiliated with the firm f_v . What is essential in the assumption (A3) is that the immediate neighbourhood of a vacancy affects how it is filled.

Let us now consider more specific functional dependencies for the probabilities P_o and P_h . First, if P_o depends only on n_v , let us denote $P_o(n_v)$, and is a decreasing function, then there is preferential attachment in the sense that a position is less likely to be opened if its holder has several other positions.

One particular functional form for P_h is obtained by assuming that P_h depends only on the distance and degree, i.e., $P_h(n_w, d_{w,v})$. If P_h is increasing in its first argument, a person holding several positions is more likely to get

the position v . Again this is a form of preferential attachment. On the other hand, if P_h is decreasing in its second argument, there is a form of peer referral; a holder of a position that is found in a close neighbourhood of the vacancy is more likely to become the new holder. In particular, when the new person is found from distance one, there is a triadic closure; the new holder of the position is known by someone in the same board with the previous holder of the position.

The first observation on the above dynamic system is that it describes a Markov chain over the neighbourhood configurations. Let $P(N^{t+1}|N^t, \dots, N^0)$ stand for the conditional probability of a configuration N^{t+1} in period $t + 1$. The Markov property means that $P(N^{t+1}|N^t, \dots, N^0) = P(N^{t+1}|N^t)$ for any $N^{t+1}, N^t, \dots, N^0 \in \mathcal{N}$. Moreover, the process is irreducible; it is possible to get from one neighbourhood configuration to any other configuration with a positive probability.

Proposition 1. *Under assumptions (A1)–(A3) the stochastic process of neighbourhood configurations is an irreducible Markov chain.*

The above result, proved in the Appendix, implies that the stochastic process over the neighbourhood configurations has a unique stationary distribution. Note that the number of possible neighbourhood configurations, although finite, can be extremely high, which means that in practice it is not the neighbourhood configurations that can be tracked but some statistics that depend on them. Irreducibility of the Markov chain implies that the time average of any statistics that depends on neighbourhood configurations converges with probability one. In particular, the time averages of distances and degrees of new board members converge.

3.2. The exponential search model

A particular functional form for the probabilities P_o and P_h can be defined by using the exponential function as in the multinomial logistic regression:

$$P_h(N(w), N(v)) = \exp[a(d_{w,v}) + b(n_w)] / C, \quad (1)$$

$$P_o(N(v)) = p^0 \exp[c(n_w)], \quad (2)$$

where $p^0 > 0$ is a constant such that $P_o(N(v)) \leq 1$, and C is a constant such that summing the probabilities over the candidates is one. To be specific, the constant C satisfies

$$C = \sum_{\substack{w \in V \\ d_{w,v} \geq 1}} \exp[a(d_{w,v}) + b(n_w)].$$

In Section 5 it is assumed that the functions in the exponential search model are

$$a(d) = -\alpha d, \tag{3}$$

$$b(n) = \beta \min\{n, K\}/K, \tag{4}$$

$$c(n) = -\gamma n, \tag{5}$$

Notice that K indicates the largest relevant number of positions. If the number of positions is larger than K , the effect of is the same as if it was K .

The model resulting from equations (3)–(5) is flexible enough for producing both realistic degree distribution and networks with small-world properties such as high clustering coefficient. In particular, a large value of β would imply a high probability of vacancies filled with people having a large number of positions, which is likely to lead to a small-world network. On the other hand, a large value of α would imply a high probability for the new person to be close to the holders of the positions in the same firm, which is likely to lead to a highly clustered network.

The exponential form in equations (1) and (2) resembles other probabilistic models for network formation such as the stochastic actor-oriented model of Snijders (2001) and exponential random graphs (Wasserman and Pattison, 1996). However, we emphasize that the exponential search model is not a specific case of either of these two families although they are applicable for two-mode networks (Koskinen and Edling, 2012; Wang et al., 2009).

4. Microeconomic Foundations

In this section we present simple microeconomic foundations for the probabilistic model that describes the filling of vacancies. It is assumed that when a firm opens a vacancy and searches for a person to fill it, there are two options: either the firm hires someone who already has a position or someone who is not holding any other position. In the former case the new person can either be found from searching through then neighbourhoods of the holders of the positions of the firm, or outside of any of the neighbourhoods.

4.1. Productivity signalling

As is typical in the search and matching literature, it is assumed that the productivity of each person is unknown but the firm gets a private signal on it. Note that in the framework of this paper, productivity can also be interpreted simply as a variable describing how suitable a person is for carrying out the task related to a vacancy. Let $s(n, d) \geq 0$ stand for the signal of the productivity of person holding $n \geq 1$ positions and located on the distance

$d \geq 1$ from the firm. Recall that n_v stands for the number of positions held by the holder of position v .

It is assumed that the payoff from hiring a person with productivity $z \geq 0$ is $u(z)$ for each firm. To be specific, u is the firm's von Neumann-Morgenstern utility function. When a firm receives a signal $s(n, d)$, the productivity z becomes a random variable with expected value $s(n, d)$ and variance determined by n and d . The signals are drawn from the productivity distribution that may depend on n and d . However, the ex-ante distribution, i.e., distribution of a signal drawn for a randomly selected person, is the same as the productivity distribution of the whole population of all holders of positions. In the extreme case when the signal is completely uninformative, the productivity distribution is the same as the distribution of productivity after the signal.

The firms' expected payoff is $\mathbb{E}[u(z)|s(n_w, d_{w,v})]$ from hiring the holder of position $w \in V$ to the vacancy v . The firm is assumed to know the distribution of signals and productivity. In practice, the firm may obtain the information through communicating with the peers of the person affiliated with the firm. As a result, a candidate may get a referral for the job.

In the simplest case, the search for a person to a vacancy is costless, which entails that the firm hires a person with the highest expected payoff. Let s^1 denote the highest signal of a person outside of holders of current positions. It is assumed that s^1 is drawn from some distribution known by the firm.

Let us briefly consider the other side of the market. It can either be assumed that people who are offered a position always accept the offer, or they can reject the offer and wait for new offers to arrive. When all the firms are similar and there is no cost of holding multiple positions at the same time, it can be assumed that a person always accept the offer. For simplicity let us assume that this is the case. Note that in the latter case, a person may find it better to reject an offer and wait for a new offer if it is likely that some "better" firm will make one in the future. In other words, when it is costly to hold many positions and firms are different, for instance larger firms pay higher salaries, there can be an option value in waiting for new offers.

Let us now return to the filling of vacancies under the assumption that offers are always accepted. In this case the firm hires a new person to v with signal s^1 outside of holders of positions in the network, when

$$\mathbb{E}[u(z)|s^1] > \max\{\mathbb{E}[u(z)|s(n_w, d_{w,v})] : w \in V\}.$$

On the other hand, the holder of position $v' \in V$ is hired if the choice v' maximizes $\mathbb{E}[u(z)|s(n_w, d_{w,v})]$ over $w \in V$ and

$$\max\{\mathbb{E}[u(z)|s(n_w, d_{w,v})] : w \in V\} \geq \mathbb{E}[u(z)|s^1].$$

If there are multiple maximizers we can assume that the new person is taken randomly among them.

The search and matching process as described above determines the probabilities P_n and P_h . To be specific, P_n and P_h are the ex ante probabilities of hiring a person outside of the network or a person who already holds one of the positions in the network, i.e., probabilities prior to a firm receiving any signal. The distributions of signals affect the properties P_h . To analyze these properties some concepts are needed. The first is a mean preserving spread. The signal s^1 is said to be mean preserving spread of s^2 if the distribution of s^1 is the same as the distribution of $s^1 + q$ where $\mathbb{E}[q|s^1] = 0$. This means that the expected value of s^1 is the same as the expected value of s^2 but the variance of s^1 is at least the same as the variance of s^2 . When s^1 is a mean preserving spread of s^2 , we can say that s^2 is a more informative signal.

The second concept that is needed is the first order stochastic dominance. The signal s^2 has the first order stochastic dominance over s^1 if there is a random variable q such that $q \leq 0$ and the inequality is strict at least in one state, and the distribution of s^1 equals the distribution of $s^2 + q$. In other words, the expected value of s^1 is lower than the expected value of s^2 while the variance of s^1 is at least the same as the variance of s^2 .

Assume that the information on candidates comes through referrals. In that case the information on candidates becomes less reliable the more there are intermediaries in the communication, which results on the signal becoming less informative when the distance increases. This property is captured in the mean preserving spread; the signal s^1 is less informative than s^2 if s^1 is a mean preserving spread of s^2 .

If the firm is risk averse and the signal becomes less informative the further away the vacancy is from the firm, then the resulting probability of a match is a decreasing function of the distance. The proof is presented in the Appendix.

Proposition 2. *Assume that $n_{v^1} = n_{v^2}$ for $v, v^1, v^2 \in V$, $v \neq v^i$, $i = 1, 2$, and $v^1 \neq v^2$. If the firm is risk averse and the distribution of $s(n_{v^1}, d_{v^1, v})$ is a mean preserving spread of the distribution of $s(n_{v^2}, d_{v^2, v})$ when $d_{v, v^1} > d_{v, v^2}$, then the probability P_h of the holder of v^1 being chosen to the vacancy v is no larger than the corresponding probability for the holder of v^2 .*

The number of positions held by a person may signal the productivity of a candidate. In particular, having more positions may signal a person's ability and hence impact positively the productivity signal. One explanation to higher productivity due to being in several boards comes from the access to more information (Mizruchi, 1996). On the other hand, there is also a mechanism acting to other direction; a high number of positions may signal

low productivity of the candidate when assigned to one more board. In either case, the first order stochastic dominance can be used in capturing the effect of number of positions ("degree") in the productivity signal. The proof of the following result can be found from the Appendix.

Proposition 3. *Assume that $d_{v^1,v} = d_{v^2,v}$ for $v, v^1, v^2 \in V$, $v \neq v^i$, $i = 1, 2$, and $v^1 \neq v^2$. If the firm is risk averse and the distribution of $s(n_{v^2}, d_{v^2,v})$ has the first order stochastic dominance over the distribution of $s(n_{v^1}, d_{v^1,v})$ when $n_{v^2} > n_{v^1}$ (alternatively when $n_{v^2} < n_{v^1}$), then the probability P_h of the holder of v^1 being chosen to the vacancy v is no larger than the corresponding probability for the holder of v^2 .*

4.2. Foundations of the exponential search model

In this section we consider the exponential search model of Section 3.2. The purpose is to give simple assumptions for firms' preferences and private information that lead to the exponential search model. As will be shown, the probability of the form (1) for filling a vacancy arises when the firms are assumed to have a common constant absolute risk aversion utility (CARA) utility function, the productivity is normally distributed, and the firms get private signals on productivities of each candidate that are type I extreme value distributed.

As before, I is the set of candidates available for a vacancy. The set of firms is F . In the following n_i stands for the number of positions held by $i \in I$, and $d_{i,k}$ stands for the distance of $i \in I$ from the firm $k \in F$. Note that $n_i = |V_{v^i}|$ where v^i is any position held by i , and $d_{i,k} = d_{v^i,v^k}$, where v^k is a vacancy that is to be filled in the firm $k \in F$.

Assume now that the expected value μ of productivity z depends on the number of positions, and the variance σ^2 depends on the distance such that

$$\mu = A + b(n_i) + \varepsilon_{i,k}, \quad (6)$$

$$\sigma^2 = -2a(d_{i,k}), \quad (7)$$

where $A \in \mathbb{R}$ is a constant and $a(d_{i,k}) \leq 0$. The term $\varepsilon_{i,k}$ reflects the private information that firm $k \in F$ has on the productivity of individual $i \in I$. These terms are not observed by an outsider but they affect the choices made by the firms. Note also that when b is an increasing function, then the expected productivity is increasing in the number of positions held by individuals. Moreover, when $a \leq 0$ is decreasing, the higher the distance the larger the variance of the productivity.

It should be observed that this model can be interpreted in terms of productivity signalling. The expected value μ is the expected value of the

productivity conditional on the number of positions, and σ^2 is its variance. In principle, the firm receives a signal on the candidates productivity as before, but now the signal has a private component $\varepsilon_{i,k}$. The main assumption is that the signalling leads to a conditional productivity distribution that depends on $\varepsilon_{i,k}$, $d_{i,k}$, and n_i with mean and variance as in (6) and (7).

The following proposition gives conditions under which the probability of choosing a candidate with a certain number of positions and distance to the firm is determined by the logistic formula of equation (1). The proof of the result is presented in the Appendix.

Proposition 4. *Assume that*

1. *the utility function of the firm $k \in F$ is the CARA utility $1 - \exp(-\gamma z)$ for $\gamma > 0$,*
2. *the productivity z is normally distributed with mean and variance as in (6) and (7), and*
3. *$\varepsilon_{i,k}$, $i \in I$, are independent identically distributed random variables from type I extreme value distribution that are known by the firm.*

It follows that the conditional probability of filling a vacancy P_h is of the form

$$P_h(n_i, d_{i,k}) = \exp[\gamma a(d_{i,k}) + b(n_i)] / C, \quad (8)$$

where $C > 0$ is the normalizing constant.

The above result is important, because it means that there is a simple structural model underlying the exponential search model. Note that the same result as above is obtained alternatively by assuming a CARA utility $1 - \exp(-z)$ and a variance $-2\gamma a(d_{i,k})$. Hence, the variance and the constant of risk aversion γ cannot be disentangled in this structural model.

Proposition 4 offers an interpretation for the parameters α and β of the model corresponding to equations (3) and (4). The parameter β is the rate at which the productivity decreases or increases as a function of the degree $|x|_K/K$. The parameter α , on the other hand, describes the rate at which the "risk" associated with the productivity signal changes when the distance of a candidate from the firm increases.

5. Fitting the Model to Data

Assume that instead of observing neighbourhood configurations there is some variable y that depends on the neighbourhood configuration and is observed at time instants $t = 1, \dots, T$. It is assumed that $y(N^t) \in \mathbb{R}^k$. For example, y could be the "degree" distribution, in which $y_i(N^t)$ equals

the number of people that are affiliated to i firms when the neighbourhood configuration is N^t .

The approach for finding the parameters that define the probabilities for filling vacancies relies on fitting the time average of variables that depend on the neighbourhood configurations into observed data. To be specific, assume that $y(N^t)$ depends on the neighbourhood configuration and an empirical realization of this statistics y is observed. In practice, y can be computed by taking the time average of variables over periods $t = 1, \dots, T$.

Assume that the parameters of the model belong to a set $Z \in \mathbb{R}$. For a given parameter value $z \in Z$, the stochastic process determined by these parameter values leads to the time average $y(z)$ over the periods $t = 1, \dots, T$. In practice this time average can be found approximately by simulating the evolution of the network from a given initial setup several times and taking the average over the simulations. The parameters can be fitted by minimizing the least squares criterion $\|y(z) - y\|^2$. Depending on the relevant properties that want to be fitted, different network dependent variables can be chosen. Let us next describe in more detail the particular criterion used in the empirical application of the following section.

5.1. Fitting the exponential search model

In this section the purpose is to explain how the exponential search model as specified in equations (3)–(5) can be fitted into empirical data. The same approach can, however, be applied for other functional forms, too. The relevant parameters of the model are $P_n, p^0, K, \alpha, \beta, \gamma$ and d^∞ . Because there are two separate stages—opening and filling of vacancies—we present a two-step procedure to fit the model. The first step concerns the opening of vacancies, i.e., the parameters p^0 and γ and the second how the vacancies are filled, i.e., the parameters α, β , and d^∞ .

From now on the number of positions that a holder of a position $v \in V$ has, will be called the degree of v . Let $V(n)$ stand for the number of positions with degree n and $V_o(n)$ the number of positions that have been opened and have had degree n . The ratio $V_o(n)/V(n)$ is the empirical frequency at which vacancies with degree n open. The parameters p_0 and γ in f_o can be obtained by fitting f_o of equation (2) into this empirical distribution.

Recall that the parameter K which appears in equation (1) of f_h indicates the largest relevant degree. Hence, this parameter can be taken as the largest degree observed in the data. Let us next describe how to obtain an estimate of the probability P_n for a new board member to be taken outside of the people currently in the boards. First, let n_o stand for the number of vacancies opened during the period of time spanned by the data. For each time period

t it is possible to find the number of individuals n_e^t who did not have any position in the previous period. The estimate for P_n is simply $\sum_t n_e^t/n_o$.

Finally, let us turn to the question of finding the remaining parameters α , β , and d^∞ . Let z stand for the vector of these parameters. Because α is related to the effect of distance in filling the vacancies and β is related to the degree, it can be argued that these parameters should be chosen such that the distance and degree distributions of new people fit empirically observed distributions. To be more specific, let $y^1(z)$ stand for the distribution of distances of newly recruited people from the position that they were chosen conditional on the person being found in some of the neighbourhoods of the vacancy that was filled. Moreover, $y^2(z)$ stands for distribution of degrees of newly recruited individuals conditional on being found among the people in the boards of the previous period.

In practice $y^1(z)$ and $y^2(z)$ are obtained by taking the time average of the simulated distributions. Let $\bar{y}^1(z)$ and $\bar{y}^2(z)$ stand for the resulting distribution when normalized into probability distributions. The corresponding empirical probability distributions are y^1 and y^2 . The criterion used in Section 6 for fitting the parameters α , β , and d^∞ of the exponential model is

$$\|\bar{y}^1(z) - y^1\|^2 + \|\bar{y}^2(z) - y^2\|^2,$$

The role of d^∞ is important for networks involving multiple connected components. For example, it affects the probability at which a company connected to a clique outside of the main component recruits new board members from the companies in the main component. In a sense, it is a virtual distance between two firms that are not connected.

5.2. Leaving vacancies empty

In practice the number of positions in companies varies. There can be several reasons for this. First, when someone leaves a position it may take some time to find a new person. Consequently, the vacancy is temporarily left empty. The second reason is that the number of positions can be increased if an interesting candidate is available. One phenomenon that can be seen in real world data is "board swapping"; a person leaves one position and is immediately given another somewhere else. For example, in the empirical application presented in next section, 4%–8% of yearly board appointments involve persons switching from one board to another.

One way to treat empty positions is simply to assume that no vacancy is left empty. The number of position in a board is taken as the maximum number of board members observed in a period of time. When simulating the model it can be assumed that all these positions are filled as if there

was somebody holding each empty seat. This means that when counting the empirical distributions for degrees, an empty position is treated as if it was held by somebody who has no other positions.

6. Empirical Application

6.1. Data

The initial data consists of 826 Finnish companies, their board members and CEOs in 2005–2015. The set of companies includes the five hundred largest companies in 2013, 2008, 2003, 1998, and the companies listed in OMX Helsinki in 2013. Due to mergers, acquisitions, and bankruptcies, there are also companies that do not exist throughout the sample period. These companies are simply excluded from the analysis.

The firms corresponding to the relevant labor market of board professionals is extracted recursively: first all the firms that are in the main component in all years are included, all firms connected to these companies in some year through interlocking board members are included, and so on. Altogether this selection consists of 484 firms and it can be considered as the pool of companies acting in the same labor market. In particular, if any of the firms in our sample has recruited a new board member, who is already in a board of some company, it is highly likely that the firm from which the new person is taken is in our sample.

The descriptive statistics of the sample of 484 companies are collected in Table 1. Note that some of the model parameters can be directly obtained from these observations. In particular, the estimated probability P_n that a new board member is an outsider is 80% is obtained by dividing $\sum_t n_e^t$ with n_0 . The probability of opening a vacancy, P_o , is about 16%, and the largest observed degree in the data is $K = 7$.

6.2. Estimated model

The solid line with circles in panel (a) of Figure 2 illustrates the distance degree of newly recruited board members when they are taken from the population of all holders of the available board positions in the network. The dotted line with triangle markers represents the distribution for a model in which the new persons are selected at random, i.e., $\alpha = \beta = \gamma = 0$ in equations (3)–(5). As can be seen, the observed distribution is considerably skewed towards small distances compared to the distribution in the random model. This indicates that there is a form of peer referral in filling the vacancies or simply a distance effect.

The degree distribution of new board members is presented in panel (b) of Figure 2. This distribution has slightly fatter tail than the corresponding

	Value
Number of firms	484
Number of positions	3731
Median number of empty positions/year	630
Total number of vacancies (n_o)	5787
New board members without a position ($\sum_t n_e^t$)	4639
New board members with a position	1148
Probability of recruiting an outsider (P_n)	80%
Probability of opening a vacancy (p^0)	16%
Largest observed degree (K)	7
Average clustering coefficient of the firm projection	0.20

Table 1: Descriptive statistics.

distribution of the random model indicating that there is a form of preferential attachment in filling the vacancies. The distributions of Figure 2 are the ones that are used in fitting the parameters α , β , and d^∞ of the exponential search model (3)–(4). The estimated parameter γ in equation (5) is close to zero (-0.005), and is therefore assumed to be zero. The estimated parameters are collected in Table 6.2. Standard errors were obtained by parametric bootstrapping; 200 samples of data were simulated, and the model was refitted to these networks.

	Value	SE	95% CI	Interpretation
α	0.72	0.06	(0.64, 0.84)	Distance parameter in equation (3)
β	1.70	0.37	(0.61, 1.82)	Degree parameter in equation (4)
d^∞	19.2	1.97	(16.8, 22.7)	Distance between unconnected positions

Table 2: Parameter estimates, standard errors (SE), and 95% confidence intervals (95% CI).

When comparing the empirical and simulated distributions, it can be seen that the fitted model performs well. The simulated distributions of distances and degrees of new board members are the gray curves in Figure 2. The non-monotonicity of the distance distribution is an interesting detail which results from the network being composed of several components that are not connected to each other in each period. In particular, firms outside of the main component are typically members of small cliques of companies, and therefore when searching for new board members among their neighbours, they tend to find them very close in the network. The parameter d^∞ plays an

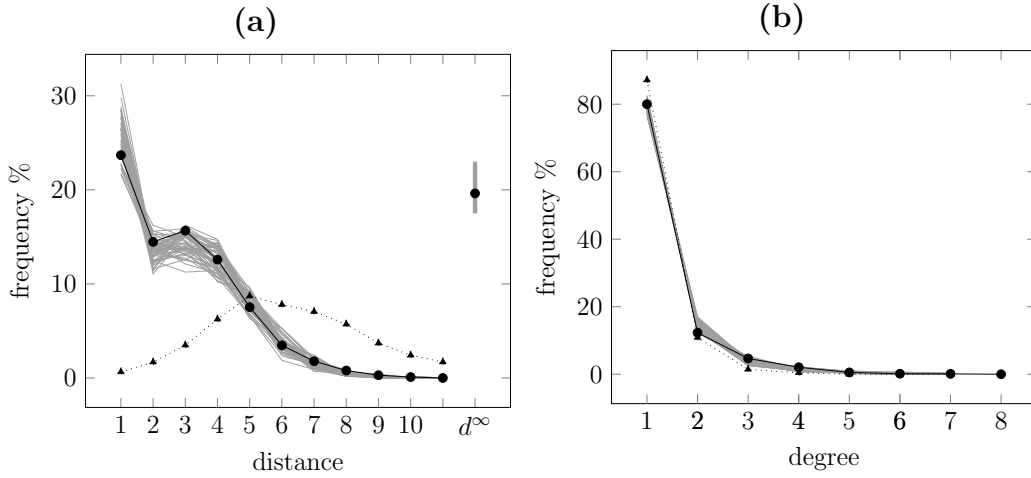


Figure 2: (a) Distance distribution of new board members (solid/black circle) and 50 simulated distributions corresponding to the fitted model (gray) and a distribution with randomly filled vacancies (dotted/gray triangle). (b) Degree distribution of new board members and 50 simulated distributions corresponding to the fitted model (gray) and a model with randomly filled vacancies (dotted).

important role for these companies, because it determines the probability of recruiting people outside of the boards in their neighbourhoods. In particular, d^∞ affects directly to the probability of person being recruited outside of the clique to which the company belongs to. Second, this parameter affects the non-monotonicity of the distance distribution. The higher the value of d^∞ the less likely it is that firms within small cliques appoint persons found in the main component, and rather appoint persons found in the same clique, i.e., within distance one. Hence, increasing d^∞ , increases the peak at distance one.

In addition to matching well to the empirical distributions over degrees and distances of new board members, the other characteristics of the fitted model are reasonably close to empirical observations. For example, when comparing the simulated degree distribution of the fitted model and the empirical distribution in terms of frequencies, they are close to each other (the square error is 0.001 for the probability distributions of degrees). The average clustering coefficient of the firm projection* of the fitted model is about 0.16, while the average over the observed networks is 0.20.

*The firm projection is the network of firms connected through common board members that is obtained from the two-mode network.

7. Conclusions

This work considers networks in which there are firms and individuals affiliated to them. New links are formed when vacancies are filled by searching people from the network. The approach taken in this work stems from the search and matching framework in labor economics. However, we consider labor markets where an individual may hold several positions at the same time, and only the firms search for new candidates and are able to direct their search to types of candidates they prefer. These features match the way how board positions in companies are filled in practice.

As shown in the paper, the search model has simple microeconomic foundations. The distance and degree of individuals in the network affect the productivity signal that the companies have on them. The further away a person is from a company, the less reliable the signal, and the higher the degree, the larger the expected productivity. These features can be interpreted in terms of peer referral and preferential attachment, respectively. It is also demonstrated that the model fits well to empirical data and is capable of explaining the formation of board interlock networks. Moreover, the empirical application shows that social networks may play a major role when firms hire new board members, and the structure of the board interlock network is shaped by the search process of new people.

There are several possible extensions of the model, such as allowing for firm heterogeneity in their practices of opening and filling vacancies. In particular, there could be a tendency to recruit new board members from same type of companies, see Currarini et al. (2009) on the role of same-type bias in a matching process of friendship network formation. Another extension would be to allow for richer dynamics by letting the number of positions vary, or letting the number of firms evolve.

Appendix A. Mathematical Proofs

Proof of Proposition 1. To prove the irreducibility it is sufficient to show that it is possible to get from one neighbourhood configuration to any other by opening and filling the vacancies. Hence, take two neighbourhood configurations N^1 and N^2 . First, by A1 and A2 there is positive probability that all the positions of N^1 are opened in the first period one and filled with the new individuals. Moreover, there is a positive probability that after filling the vacancies with new individuals, exactly the ones which have holders with multiple positions in N^2 are opened (A1) in the second period. For each vacancy it is possible that any of the holder of other positions becomes the new holder by (A3). Hence, there is a positive probability that the new holders

have exactly the same set of positions that the holders of the positions in N^2 . \square

Proof of Proposition 2. The productivity is $z^j = s^j + \varepsilon^j$, $j = 1, 2$, for signals $s^1 = s(n_{v^1}, d_{v^1, v})$ and $s^2 = s(n_{v^2}, d_{v^2, v})$. The terms ε^j , $j = 1, 2$, are mean zero error terms. The ex-ante distributions of s^1 and s^2 are the same, while ε^1 is a mean preserving spread of ε^2 by the assumption of the proposition.

Consider the functions $f^j(s) = \mathbb{E}_{\varepsilon^j}[u(z)|s]$, $j = 1, 2$, i.e., the expected utilities for a given signal s and mean zero error terms ε^1 and ε^2 . Because the firm is risk averse, it holds that $f^2(s) \geq f^1(s)$. The probability that the holder of v^j is better than the holder of another position is the probability that $f^j(s)$ is higher than \bar{u} which corresponds to the expected value of the other position. Because the ex-ante distributions of signals are the same and $f^2(s) \geq f^1(s)$, it follows that the probability of $f^2(s) \geq \bar{u}$ is no smaller than the probability of $f^1(s) \geq \bar{u}$. Hence, the result follows. \square

Proof of Proposition 3. To suppress the notation let us denote the signals by $s^1 = s(n_{v^1}, d_{v^1, v})$ and $s^2 = s(n_{v^2}, d_{v^2, v})$. Recall that once the signal is realized the productivity is $z^j = s^j + \varepsilon^j$, $j = 1, 2$. Given that the ex-ante distributions of signals are the same, the first order stochastic dominance implies that ε^2 has the first order stochastic dominance over ε^1 . By definition this means that for any increasing u it holds that $f^2(s) \geq f^1(s)$ for all s , where $f^j(s) = \mathbb{E}_{\varepsilon^j}[u(z)|s]$, $j = 1, 2$. As in the proof of Proposition 2 it follows that the probability of choosing the holder of v^1 is no larger than the corresponding probability for the holder of v^2 . \square

Proof of Proposition 4. Taking the expected value of the CARA utility over the normal distribution with mean $\mu = A + b(n_i) + \varepsilon_{i,k}$ and variance $\sigma^2 = -2a(d_{i,k})$ yields

$$1 - \exp(-\gamma\mu + \gamma^2\sigma^2/2) = 1 - \exp[\gamma(-A - b(n_i) - \varepsilon_{i,k}) - \gamma^2a(d_{i,k})].$$

Maximizing this function is equivalent to maximizing

$$\gamma a(d_{i,k}) + b(n_i) + \varepsilon_{i,k}.$$

Because $\varepsilon_{i,k}$, $k \in F$ and $i \in I$, are type I extreme value distributed, the ex ante probability of choosing a candidate is determined by the logit expression (8) for the conditional choice probabilities, see, e.g., McFadden (1981). \square

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