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Self-control problems and personalized regulation: Sin licenses revisited

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ABSTRACT

We analyze personalized regulation in the form of sin licenses to correct the distortion in the consumption of a harmful good when consumers suffer from varying degrees of self-control problems. We take into account preference uncertainty, which generates a trade-off between flexibility and commitment provided by sin licenses. We also account for the possibility that consumers may trade the sin good in a secondary market, which partially erodes the commitment power of sin licenses. We show that if sophisticated consumers are allowed to choose any general, individualized pricing scheme for sin goods, they will choose a system of sin licenses. Nevertheless, sin licenses do not implement the social optimum in our general setting. We derive a simple criterion for assessing whether switching to a system of sin licenses improves welfare over linear sin taxes.

JEL Classification: H21, H30, I18

Keywords: self-control problems, sin licenses, non-linear pricing, demand uncertainty, secondary markets

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1 Introduction

Literature in behavioral economics suggests that consumers sometimes make mistakes. A prominent example is excessive consumption of harmful goods such as alcohol, tobacco and unhealthy food, which may be caused for example by self-control problems. Excessive consumption may provide a rationale for regulation, which typically takes the form of linear sin taxes (O'Donoghue and Rabin 2003,2006; Gruber and Köszegi 2004; Haavio and Kotakorpi 2011). Individuals with self-control problems may value sin taxes as a commitment device to lower consumption. However, when people differ in their degree of self-control problems, linear taxes cannot achieve the first-best outcome: a tax based on some measure of average self-control problems distorts the consumption of individuals without a self-control problem and will be too low for individuals with severe self-control problems.

The question then arises, whether one could do better by personalized regulation. We reconsider the use of so called sin licenses, first suggested by O'Donoghue and Rabin (2003), that allow individuals with self-control problems to commit to a given level of future consumption. We analyze generalized sin licenses, where each consumer chooses a maximum quota of sin goods that he can purchase in the future subject to a lower tax (zero rate under pure sin licenses), whereas purchases exceeding the quota will incur a higher tax (infinite rate under pure sin licenses). We account for preference uncertainty: the consumer may not know for sure his true future consumption needs. Hence there is a trade-off between commitment and flexibility provided by regulation. We also account for the possibility of trading the sin good in a secondary market, which may limit the scope for personalized regulation.

In general, the difficulties with implementing non-linear taxation are well-known. Our analysis is motivated by the notion that regulating harmful consumption may be a special case: the very time-inconsistency that leads to the need for regulation, could be exploited to implement optimal regulation. This idea is expressed for example in O'Donoghue and Rabin (2007): "If our goal is to implement a policy that combats present bias, but we are worried that this policy might hurt people who don't have present bias, why not let people voluntarily select in advance whether to be subject to the policy. If everyone were fully sophisticated, such a scheme can be very effective, because we can count on all agents to choose whatever incentives are best for them."

Our first key result shows that if sophisticated[1] individuals with self-control problems are allowed to choose *any* non-linear personalized scheme for regulating their future sin goods consumption, they will in fact choose a scheme of sin licenses. An important part of the intuition is that sin licenses provide a cheap way of achieving commitment to a given level of consumption. This result provides a rationale for focusing on sin licenses among all possible personalized schemes.

[1] Sophistication (awareness) and naivete (complete unawareness) regarding one's self-control problem were discussed by Strotz (1955-6) and Pollak (1968). O'Donoghue and Rabin (1999) analyze the implications of sophistication and naivete, and O'Donoghue and Rabin (2001) analyze the intermediate case of partial naivete.

However, we show that even though sin licenses are preferred by sophisticated consumers, they are not socially optimal. The intuition is that individuals would like to achieve (some) commitment at the lowest possible cost (to themselves), but the costs (at the individual level) are irrelevant from a social point of view, as revenue collected by the regulating scheme can be redistributed back to consumers in a lump-sum fashion. Hence the objectives of the social planner and the consumer diverge, even when consumers are sophisticated. This result is in contrast to O'Donoghue and Rabin (2005), who show that under some stylized settings - namely when there is no preference uncertainty and no secondary market trade - sin licenses achieve the first-best outcome.

Given that neither linear taxes nor sin licenses achieve the first-best - and given that the socially optimal regulating mechanism is likely not implementable - how should one choose which type of regulation to use? We derive a simple sufficient condition under which sin licenses improve welfare over linear taxation, in a society consisting of sophisticated individuals: sin licenses improve welfare if the reform does not lead to a reduction in tax revenue. If this criterion is met, we can infer that the consumers' choices of license quotas were on the whole not determined by the motivation to minimize costs, but rather by the motivation to minimize distortions.

Further, taking into account that some individuals are naïve, the policy conclusions from our analysis are two-fold. First, *replacing* a linear sin tax with a system of sin licenses is not a good idea: this would reduce the welfare of naives and might also reduce the welfare of sophisticates. Naives see no value in constraining their future choices, and would opt for the least stringent possible scheme. Second, however, welfare may be improved by *supplementing* the current linear tax with a voluntary system of sin licenses that would allow individuals to opt for *stricter* personalized regulation. In this sense, sin licenses and sin taxes should be regarded as complements rather than substitutes. The aim of this type of a reform is to target sophisticated consumers, while naive consumers stick to the original linear tax. The reform is again welfare improving if tax revenue does not decline.

We also show that sin licenses are more likely to produce welfare gains if people differ a lot in the severity of self-control problems. Quite intuitively, in such a situation there is more need for personalized regulation. Secondary market trade, on the other hand, constrains the scope for personalized regulation, but is unlikely to undermine it altogether.

Regarding the trade-off between commitment and flexibility, our paper is related to Amador, Werning and Angeletos (2006), who study a sophisticated individual's choice of how to regulate the level of savings, when the individual suffers from self-control problems.[2] Self-control problems may lead to too high current consumption and inadequate savings. Amador et al. show that in certain circumstances, sophisticated consumers would opt for a minimum-savings policy. Since a floor on savings is equivalent to a ceiling on current con-

[2] Another related paper is Galperti (2014), who considers the trade-off between commitment and flexibility when a monopolist and/or social planner attempts to provide mechanisms that optimally screen consumers with varying degrees of self-control problems.

sumption, their result is closely related to ours: in its simplest form, a scheme of sin licenses implies an upper limit on sin goods consumption. Our paper differs from Amador et al. in that we analyze the relationship between the individual's preferred policy and the social optimum, and show that in general they do not coincide. This finding also motivates our comparison between the merits of voluntary personalized regulation *vs.* mandatory uniform regulation.

A related literature studies attempts to achieve self-control in the market (Heidhues and Köszegi 2009, DellaVigna and Malmendier 2006, Köszegi 2005). Market mechanisms have the advantage of being voluntary and personalized, but there are two caveats: market mechanisms in general do not work for naives; and they may be ineffective in achieving commitment. For example, in a competitive market, a consumer may reach a contract with one firm to limit the supply of harmful goods, but another firm will have an incentive to supply the good at marginal cost (Köszegi 2005, Gottlieb 2008). Firms' incentives to take advantage of consumers' self-control problems have been analyzed by Heidhues and Köszegi (2010) and Eliaz and Spiegler (2006). Sin licenses can be viewed as an attempt to combine the positive sides of market mechanisms (voluntary, personalized regulation) and government intervention (wider scope and better commitment, since public policy cannot be changed over night).[3]

The questions we study also bear resemblance to the analysis of price vs. quantity regulation in environmental economics, when the regulator does not know the abatement costs of polluting firms - the classic reference here is Weitzman (1974). Sin licenses are to a certain extent akin to pollution quotas, whereas sin taxes resemble pollution taxes. However, there is a crucial difference. In the environmental economics context, it is typically assumed that a social welfare maximizing regulator designs the regulating mechanism for the polluting firm. Indeed, there are no incentives for self-regulation in the case of externalities. In our application, the consumer designs the mechanism for himself.[4]

The paper proceeds as follows. The model is introduced in Section 2. The properties of sin licenses in the presence of preference uncertainty and secondary market trade are examined in Section 3. Section 4 shows that if consumers are allowed to choose any personalized non-linear pricing scheme for sin goods, they (under certain conditions) opt for a system of sin licenses. Section 5 analyzes the role of sin licenses in regulating harmful consumption as a substitute or complement to linear taxation. Section 6 concludes with a discussion of the policy implications of our analysis.

[3]For a review of issues related to the use of commitment devices, see Bryan, Karlan and Nelson (2010). Bhattacharya and Lakdawalla (2004) discuss smoking licenses, and Beshears et al. (2005) discuss related schemes involving prospective choices on the part of consumers.

[4]The information hierarchy, or the asymmetric information setting, is also more complex in the situation we study. In the environmental economics application, there are two layers: the firm knows the abatement costs, while the regulator does not. In our setting, there are three layers: the ex post self knows the ex post preferences, including the realization of the preference shock, but suffers from present bias; the ex ante self knows the ex ante preferences, and the severity of self-control problem, but he does not know the realization of the preference shock; the regulator knows neither of these. The (ex ante) consumer, who knows more about himself than the regulator does, is allowed to design the personalized regulation mechanism.

2 The model

We consider a model where consumers have a quasi-hyperbolic discount function (Laibson 1997), using a set-up that is similar for example to O'Donoghue and Rabin (2003; 2006). In the model, consumers suffer from varying degrees of self-control problems. Life-time utility of an individual (i) is given by

$$U_{it} = (u_{it}, ..., u_{iT}) = u_{it} + \beta_i \sum_{s=t+1}^{T} \delta_i^{s-t} u_{is}, \tag{1}$$

where $\beta_i, \delta_i \in (0, 1)$ and u_{ti} is the periodic utility function. We assume that the quasi-hyperbolic discount factor β has a cumulative distribution function $M(\beta)$ over some support $[\beta_L, \beta_H]$, with $0 \leq \beta_L < \beta_H \leq 1$. Quasi-hyperbolic discounting implies that preferences are time-inconsistent: discounting is heavier between today and tomorrow, than any two periods that are both in the future.

We assume that utility is quasilinear with respect to a composite good (z). Consumer utility is also affected by the consumption of another good (x), which is harmful in the sense that it yields positive utility in the short run, but has some negative effects in the long run. Specifically, we assume that periodic utility is given by

$$u_{it}\left(x_{it}, x_{i,t-1}, z_{it}\right) = \theta_{it} v_i(x_{it}) - h_i\left(x_{i,t-1}\right) + z_{it}, \tag{2}$$

where $v' > 0, v'' < 0$ and the harm function is characterized by $h' > 0$ and $h'' \geqslant 0$. We allow individuals to differ in their preferences for the sin good, v_i, as well as in terms of the harm function h_i. θ_{it} is an individual-specific preference shock that is realized in period t. The model of individual preferences corresponds to O'Donoghue and Rabin (2006), except that we allow for preference uncertainty.[5]

We assume that there is no borrowing or lending. Given this assumption and our specification for the periodic utility function in (2), in each period t an agent whose objective is to maximize (1) chooses x_t so as to maximize $u_i(x_t) = \theta_{it} v_i(x_t) - \beta_i \delta h_i\left(x_t\right) + z_t$. Maximization is subject to a per-period budget constraint $s x_t + z_t \leq B + \Pi$. We assume that product markets are competitive and normalize the producer price to 1, and $s = 1 + \tau$ denotes the consumer price of good x, where τ is a possible per unit tax on good x. B is the consumer's income (taken to be exogenous) and Π is a possible lump-sum subsidy received by the consumer from the government. Taxes and subsidies will be modelled in more detail in later sections. Given the above specification, the demand for good x, given consumer price s satisfies

[5] As in O'Donoghue and Rabin (2006), we assume that the marginal benefits and marginal costs of consumption are independent of past consumption levels. In such a setting, it is not essential that the harm is modelled as occuring only in the period following consumption - h can be thought of as the discounted sum of harm occurring in all future periods. See Gruber and Köszegi (2004) for an analysis where past consumption affects current marginal utility.

$$\theta_{it} v_i'(x_i^*(\theta_{ti}, \beta_i; s)) - \beta_i \delta_i h_i'(x_i^*(\theta_{it}, \beta_i; s)) = s. \tag{3}$$

In particular, under laissez-faire ($\tau = 0$) we have

$$\theta_{it} v_i'(x_i^*(\theta_{ti}, \beta_i; 1)) - \beta_i \delta_i h_i'(x_i^*(\theta_{ti}, \beta_i; 1)) = 1. \tag{4}$$

However, the time-inconsistency in preferences implies that the consumer would like to change his behavior in the future: Maximizing (1) from the next period onwards would amount to maximizing $u_i^o(x_t) = \theta_{ti} v(x_t) - \delta_i h_i(x_t) + z_t$ each period. (See equation (1) and think of a consumer in period t, making consumption decisions for period $t+1$ onwards.) Therefore, when thinking about future decisions, the consumer would like to choose consumption levels that maximize $u_i^o(x_t)$.

In general the issue of how to conduct welfare analysis when consumers have time-inconsistent preferences is far from straight-forward (e.g. Bernheim and Rangel 2009). We follow Gruber and Köszegi (2004) and O'Donoghue and Rabin (2003, 2006) and take the so-called "long-run criterion" as the appropriate welfare criterion - that is, we take the utility function $u_i^o(x_i)$ to be the one that is relevant for welfare evaluation. This is a natural choice in our setting, since we assume that regulation is implemented from the period after the policy decision is made. Therefore, consumers consistently agree that $u^o(x)$ is the relevant utility function from the point of view of making regulatory policy.

Given the above assumptions, the optimal level of consumption $x_i^o(\theta_{ti})$ satisfies

$$\theta_i v_i'(x_i^o(\theta_{ti})) - \delta_i h_i'(x_i^o(\theta_{ti})) = 1. \tag{5}$$

Because of quasi-hyperbolic discounting ($\beta_i < 1$), the laissez-faire equilibrium level of consumption of the harmful good ($x_i^*(\theta_{ti}, \beta_i; 1)$) is higher than the optimal level of consumption ($x_i^o(\theta_{ti})$).

3 Sin licenses with preference uncertainty and (potential) secondary market trade

3.1 Preference uncertainty

We consider a continuous-demand version of O'Donoghue and Rabin's system of sin licenses (O'Donoghue and Rabin 2005), where consumers decide in period $t-1$ on a quota of sin licenses (y) to be used for consumption in period t. Sin licenses are given out for free. Purchases without a license incur a per-unit tax τ_2, whereas purchases with a license are subject to a reduced tax τ_1. The simplest case is the one involving *pure sin licenses* (as in O'Donoghue and Rabin 2005), where $\tau_1 = 0$ while τ_2 is prohibitively high (implying that no one actually buys sin goods without a license). We analyze *generalized sin licenses* where τ_1

may be above zero. Further, unlike in O'Donoghue and Rabin, the possibility of secondary market trade puts an upper bound on τ_2 (but nevertheless $\tau_1 < \tau_2$).

In the current subsection, we analyze generalized sin licenses in the presence of preference uncertainty, and introduce the trade-off between flexibility and commitment. This trade-off is a crucial feature associated with a system of sin licenses - or any other commitment device - in the presence of demand uncertainty. To keep the analysis as simple as possible, we assume for now that purchases without a license are subject to a prohibitively high tax τ_2. The implications of potential secondary market trade (putting a cap on τ_2) are deferred to the next subsection. In Sections 3 - 5.2, we assume that consumers are sophisticated, that is, they are fully aware of their self-control problem. In Section 5.3, we discuss policy conclusions when some individuals are naive.

Assume that the preference shock θ_{it} has a cumulative distribution function $F_i(\theta_i)$ over some support $\left[\underline{\theta}_i, \overline{\theta}_i\right]$. The period t preference shock is realized in that period. In the previous period, when the amount of sin licenses is chosen, the consumer only knows the distribution of θ_{it}. Ex ante, individuals make decisions concerning next-period sin licenses according to their long-run utility function. Expected (long-run) indirect utility, given the amount of sin licenses demanded, y, is

$$V(y) = E_\theta\left[\theta v(x(\theta,\beta)) - \delta h(x(\theta,\beta)) - px(\theta,\beta)\right]$$

where $p = 1 + \tau_1$,

$$x(\theta,\beta) = \min\left\{y, x^*(\theta,\beta;p)\right\}$$

and $x^*(\theta,\beta;p)$ is given by (3) . The actual consumption level $x(\theta,\beta)$ is chosen ex post, when the realization of the preference shock θ is known. The ex post consumption choice is influenced by self-control problems (if $\beta < 1$). Through the system of sin licenses, the consumer can attempt to influence his next-period consumption choices ex ante.

If the preference shock realization is small, $\theta \leq \theta_1(y;\beta)$, the realized consumption falls short of the consumer's quota of sin licenses $x(\theta,\beta) = x^*(\theta,\beta;p) < y$. On the other hand, if the shock realization is large $\theta > \theta_1(y;\beta)$, the consumer uses the entire quota of sin licenses $x(\theta,\beta) = y$. The critical value $\theta_1(y;\beta)$ is given by

$$\theta_1(y,\beta;p) = \frac{\beta\delta h'(y) + p}{v'(y)}. \tag{6}$$

In online Appendix B 1 we also show that consumer's optimal choice of sin license quota is characterized by the first-order condition

$$E\left[\theta \mid \theta \geq \theta_1\right] v'(y) - \delta h'(y) - p = 0.$$

Denote the rational (i.e. optimal from the ex ante consumer's perspective) level of consumption, with preferences θ and consumer price p, by $x^o(\theta;p)$. We show in online Appendix

B 1 that fully rational consumers, with $\beta = 1$, choose $y^* (\beta = 1) \geq x^o \left(\overline{\theta}; p\right)$. For a rational consumer, there is no need to ex ante constrain the ex post choices, and in equilibrium $x(\theta, \beta = 1; p) = x^o(\theta; p)$ for all θ. Then in particular pure sin licenses (with $\tau_1 = 0$ and hence $p = 1$), unlike linear taxes, have the desirable property that they do not distort the decisions of fully rational individuals. This is related to the idea of asymmetric paternalism discussed for example by Camerer et al. (2003).

Consumers with self control problems ($\beta < 1$), on the other hand, choose $y^* (\beta) < x^o \left(\overline{\theta}; p\right)$ and sin licenses do not in general implement the first-best for them. For consumers with self-control problems, sin licenses imply a trade-off between flexibility in the face of preference shocks vs. commitment in the face of self-control problems. On the one hand, the consumer would like to consume the (ex ante) optimal amount even when the preference shock is large. This argument favors flexibility, and hence acquiring a large quota of sin licenses. On the other hand, however, high consumption may arise due to self-control problems, rather than preferences. This argument favors commitment, and acquiring a smaller amount of sin licenses.

In particular, with pure sin licenses ($\tau_1 = 0, p = 1$) realized consumption is at the laissez-faire level (and hence too high for consumers with $\beta < 1$) for low levels of the preference shock, $\theta < \theta_1$. On the other hand, for $\theta > \theta_1$, $x(\theta, \beta)$ is constant at $x(\theta, \beta) = y$. Hence sin licenses imply excessive flexibility for low preference shock realizations, and excessive commitment (no flexibility in the face of preference shocks) for high preference shock realizations.

3.2 (Potential) secondary market trade

We next turn to the implications of potential secondary market trade, while continuing also to take into account demand uncertainty. As in the previous subsection, each unit of sin licenses allows consumers to buy one unit of the sin good at price p. The price p includes a possible tax $\tau_1 \geq 0$, so that $p = 1 + \tau_1$. In contrast to the previous section, we now assume that without sin licenses, the consumer can buy sin goods at the (per unit) price $q = 1 + \tau_2 > p$. Depending on circumstances (e.g. the specifics of legislation and regulation) this trade without a license can take place in either primary (official) markets or in secondary (black) markets.

A system of personalized regulation is inherently vulnerable to potential secondary market trade: consumers would ex post have incentives to create a secondary market, where individuals subject to more stringent regulation (high tax) buy the sin good from individuals subject to less stringent regulation (low tax). Hence there are potentially profitable transactions to be made if e.g. rational individuals hoarde sin licenses, and re-sell them to consumers with self-control problems ex post. The emergence and properties of secondary market trade are anayzed in detail in an early working paper version of this paper (Haavio and Kotakorpi 2012).

We assume that secondary market trade involves a transaction cost k per unit of goods traded. If the government were to allow consumers without sin licenses to buy the good at price $q \leq p+k$, any secondary market activity would be eliminated. The government then collects the price difference $\kappa \equiv q-p$ in the form of a sin tax $\tau_2 = \tau_1 + \kappa$; i.e. τ_2 is now set at a level that does not prohibit consumption without a sin license, but nevertheless $\tau_2 > \tau_1$ so that a license allows purchases at a reduced tax. If this form of primary market trade without a sin license (at price $q \leq p+k$) is not allowed, the transactions may take place in the secondary market at price $p+k$. In sum, the potential for secondary market trade caps the price difference relating to purchases with or without a license, so that $q-p \leq k$.

When secondary market trade is possible, the consumer's expected indirect utility, with a given amount of sin licenses y is

$$V(y) = E_\theta \left[\theta v\left(x\left(\theta,\beta\right)\right) - \delta h\left(x\left(\theta,\beta\right)\right) - px\left(\theta,\beta\right) - (q-p)x^q\left(\theta,\beta\right)\right]$$

where

$$x^q\left(\theta,\beta\right) = \max\left\{0, x\left(\theta,\beta\right) - y\right\}$$

is the amount of sin goods bought without a license at price q.

Realized consumption $x\left(\theta;\beta\right)$ is characterized by the following expressions

$$x\left(\theta;\beta\right) = \begin{cases} x^*\left(\theta,\beta;p\right) & \text{if } \theta < \theta_1\left(y,\beta;p\right) \\ y & \text{if } \theta_1\left(y,\beta;p\right) < \theta < \theta_2\left(y,\beta;q\right) \\ x^*\left(\theta,\beta;q\right) & \text{if } \theta > \theta_2\left(y,\beta;q\right) \end{cases}$$

where $x^*\left(\theta,\beta;p\right)$ is given by (3), $\theta_1\left(y,\beta;p\right)$ is given by (6), $x^*\left(\theta,\beta;q\right)$ is characterized by

$$\theta v'\left(x^*\left(\theta,\beta;q\right)\right) - \beta\delta h'\left(x^*\left(\theta,\beta,q\right)\right) - q = 0$$

and $\theta_2\left(y,\beta;q\right)$ is given by

$$\theta_2\left(y,\beta;q\right) = \frac{\beta\delta h'\left(y\right)+q}{v'\left(y\right)}. \tag{7}$$

In online Appendix B 2 we show that the consumer's optimal choice of sin license quota is characterized by the first-order condition

$$\begin{gathered}\left[F\left(\theta_2\right) - F\left(\theta_1\right)\right]\left\{E\left[\theta \mid \theta_2 \geq \theta \geq \theta_1\right]v'\left(y\right) - \delta h'\left(y\right) - p\right\} \\ +\left[1-F\left(\theta_2\right)\right]\left(q-p\right) = 0.\end{gathered} \tag{8}$$

Notice in particular, that the quota of sin licenses y now determines consumption only at the medium range of preference shock realizations $\theta_1\left(y,\beta;p\right) < \theta < \theta_2\left(y,\beta;q\right)$ - this is captured by the terms on the first row of the first-order condition (8). As in the previous subsection, with small shocks ($\theta < \theta_1$), realized consumption falls short of the quota. On the other hand, now that consumption without a license is not prohibited, the consumer yields to the temptation of purchasing more of the good ex post, if the preference shock realization is high ($\theta > \theta_2$). The consumer pays the higher price q for the purchases exceeding the quota, implying higher monetary costs; also these considerations affect the choice of the quota, as captured by the term on the second row of (8). However, if q is so high that $\theta_2(y, \beta; q) > \overline{\theta}$ the consumer is never tempted by the secondary market. Then the results stated in Section 3.1 apply.

A key point to note is that the presence of (potential) secondary market trade alters the trade-off between flexibility and commitment associated with sin licenses. The possibility of secondary market trade partially erodes the commitment-power of sin licenses: the quota y does not constitute a binding upper bound for consumption. On the other hand, there is more flexibility in the face of preference uncertainty: since purchases without a license are possible, at a higher price q, the mechanism now allows the consumer to react to true consumption needs (θ) also at the high end of preference shock realizations ($\theta > \theta_2$).

4 General non-linear pricing scheme

In the previous section, the commitment scheme was constrained to be of a particular, simple type: a lower tax τ_1 up to a quota y of the sin good, and a higher tax τ_2 thereafter. In the current section, we consider regulating sin goods via a completely general, personalized, non-linear pricing scheme. As in the case of sin licenses, the idea is that there is voluntary self-selection, i.e. the consumer himself chooses in period t the pricing scheme to be applied to his purchases of the sin good in period $t+1$. That is, the consumer self-selects what type of regulation should be applied to his purchases of the sin good in the future.

Assume that the consumer chooses (ex ante) a general non-linear pricing scheme $T(x)$, where $T(x)$ is the total price for x units of the sin good. This scheme will be applied to sin good purchases in the next period. The scheme is therefore again chosen so as to maximize expected long-run indirect utility

$$E_\theta [V(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} V(\theta) f(\theta) \, d\theta \tag{9}$$

where $f(\theta)$ is the density function of θ and[6]

$$V(\theta) = \theta v(x(\theta)) - \delta h(x(\theta)) - T(x(\theta)) + B + \Pi \tag{10}$$

Given the pricing scheme $T(x)$, the quantity of sin goods is chosen ex post to maximize

$$\widehat{V}\left(\widehat{\theta}, \theta; \beta\right) = \theta v\left(x\left(\widehat{\theta}\right)\right) - \beta\delta h\left(x\left(\widehat{\theta}\right)\right) - T\left(x\left(\widehat{\theta}\right)\right) + B + \Pi \tag{11}$$

where $x\left(\widehat{\theta}\right)$ is the consumption level intended for ex post type $\widehat{\theta}$. We assume that the pricing

[6]Clearly, x also depends on the degree of self-control problems β, i.e. $x(\theta; \beta)$, but to simplify notation, we have left β out of the formulas.

scheme has to satisfy the following constraints:

$$T'(x) \geq p \tag{12}$$

where the price floor $p = 1 + \tau_1 \geq 1$ (the sin good is not subsidized, and it may be subject to a minimum tax τ_1) and

$$T'(x) \leq q \tag{13}$$

where the price ceiling $q = 1 + \tau_2 \leq p + k$ (at each point, the per unit price has to be no bigger than the secondary market price $p + k$, however the government can choose a lower ceiling). The system parameters p and q are set by the government, but the consumer can choose any personalized non-linear pricing scheme where the marginal price is within these bounds. Further, assume that the revenues from the pricing scheme are redistributed back to consumers via uniform lump-sum subsidies.

4.1 The pricing scheme chosen by the consumer

We derive the conditions characterizing the consumers's optimal choice of $T(x)$ in Appendix A 1. The main result of this analysis is summarized in the following Proposition:

Proposition 1 *Assume that consumers are sophisticated and the distribution of the preference shock θ is such that the hazard rate $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is non-decreasing. Then a system of sin licenses implements the consumers' preferred personalized non-linear pricing scheme.*

Proof. See Appendix A 1. ■

The proposition shows that sin licenses have the interesting property that they would be chosen by sophisticated consumers among all possible, completely general and individualized non-linear pricing schemes for sin goods. The solution is therefore, as in the previous section, characterized by excessive flexibility (laissez-faire consumption, if $p = 1$) at low levels of the preference shock ($\theta < \theta_1$), combined with excessive *in*flexibility, at higher level of the preference shock: the solution to the non-linear pricing problem is a bunching equilibrium where the individual consumes the same amount $x(\theta) = y$ at all shock realizations $\theta \in [\theta_1, \theta_2]$. Further, the possibility of secondary market trade undermines the commitment power of personalized control (implemented by sin licenses) at high preference shock realizations, as consumers with $\theta > \theta_2$ are tempted by the secondary market.

The key to the intuition behind Proposition 1 is to note that in choosing the regulating scheme, consumers have two objectives: to minimize the monetary costs of regulation, while at the same time reducing distortions in consumption. Pure sin licenses, where consumption of the sin good is subject to no tax up to the binding quota y, provide the cheapest possible means of achieving (at least some level of) commitment: the consumer always pays the minimum price for the sin goods he buys. More generally, even with the possibility of secondary market trade, a scheme of sin licenses combine minimum monetary costs at low levels of consumption with maximum feasible commitment at higher levels of consumption.

The result stated in Proposition 1 hinges on the hazard rate $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$. To understand this property, note that for the ex ante consumer, $\lambda(\theta)$ is essentially a benefit-cost ratio of personalized control. Tighter personalized control, in the form of a higher marginal price $T'(x(\theta))$, alleviates distortions, or internalities, at consumption level $x(\theta)$, which has the frequency, or probability mass, $f(\theta)$. On the other hand, the downside is higher monetary costs $T\left(x\left(\theta'\right)\right)$ and/or decreased flexibility at all higher realizations of the preference shock $\theta' > \theta$, which have the frequency, or probability mass, $1 - F(\theta)$. The ratio of benefits and costs is then (proportional to) $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$.

The situation is a mirror image of a typical non-linear taxation problem: the basic logic is similar, but turned upside down. In the non-linear taxation problem, the objective is to extract large payments - i.e. tax revenue - with minimal (distorting) behavioral effects (on labor supply); then $\lambda(\theta)$ is a *cost-benefit* ratio, and a high value of $\lambda(\theta)$ calls for a *low* marginal income tax (at income bracket θ). In our application, the consumer's objective is to induce large (corrective) behavioral effects, combined with minimal payments: $\lambda(\theta)$ is a *benefit-cost* ratio, and a high value of $\lambda(\theta)$ calls for a *high* marginal price $T'(x(\theta))$.

With many plausible distributions of θ, the hazard rate is indeed increasing (or non-decreasing) in θ. One simple example is the uniform distribution. More generally, any distribution which has a finite upper bound $\overline{\theta}$, has the property that $\lambda'(\theta) > 0$ at least at the upper end of the distribution (since $\lim_{\theta\to\overline{\theta}} \lambda(\theta) = \frac{f(\theta)}{1-F(\theta)} = \infty$ if $f\left(\overline{\theta}\right) > 0$, while $\lim_{\theta\to\overline{\theta}} \lambda(\theta) = -\frac{f'(\theta)}{f(\theta)} = \infty$ if $f\left(\overline{\theta}\right) = 0$ but $f(\theta) > 0$ for all $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$). Intuitively, tight personalized control applied only at higher levels of consumption is unlikely to significantly increase expected monetary costs. Then it is rather natural that the consumer prefers a system that combines i) flexibility and low monetary costs at low levels of consumption, with ii) commitment at high levels of consumption. The system of sin licenses has these characteristics.

Also the bunching property of the equilibrium ($x(\theta) = y$ for $\theta \in [\theta_1, \theta_2]$) can be explained by referring to the benefit-cost ratio $\lambda(\theta)$. An increasing benefit-cost ratio $\lambda(\theta)$ of personalized control would ideally call for a consumption scheme that is at least partially decreasing in θ. But a scheme where realized consumption should depend negatively on the need to consume (θ) cannot be implemented. If the scheme stipulates that $x\left(\theta'\right) < x(\theta)$ for $\theta' > \theta$, ex post, the consumer with high consumption needs (preference shock θ') will pick the higher consumption level $x(\theta)$, rather than the lower consumption level $x\left(\theta'\right)$, stipulated by the putative scheme. The solution to this problem is bunching: If the decreasing consumption scheme cannot be implemented, the best alternative (ex ante) is to implement a constant consumption scheme. The system of sin licenses has also this feature.

Finally, Proposition 1 implies the following Corollary, which shows that if the choice of the regulating mechanism is left to the consumers themselves, they will prefer cases where the gap between the price floor (p) and the price ceiling (q) in the sin licensing scheme is as large as possible. In this sense, pure sin licenses where $p = 1$ are indeed an interesting special case.

Corollary 2 *Assume that $\lambda'(\theta) \geq 0$. a) Assume that the consumer can choose between i) a linear price $s = 1 + \tau$ (where $\tau > 0$ is a sin tax), and ii) a system of sin licenses with $p < s < q$. The consumer strictly prefers the system of sin licenses. b) Assume that the consumer can choose between two systems of sin licenses, i) (p_1, q_1) and ii) (p_2, q_2), where $p_2 < p_1$ and $q_2 > q_1$. The consumer prefers the system (p_2, q_2), with the lower floor (p), higher ceiling (q) and therefore a wider corridor $(q - p \equiv \kappa)$. c) If the pure system of sin licenses, $p = 1$, $q = \infty$, is feasible (it is not undermined by secondary markets), the consumer (weakly) prefers this system to any other (linear or non-linear) pricing system.*

Proof. a) and b). The results follow from the proof of Proposition 1. In both cases (a and b), the consumer could choose pricing system i), but he chooses system ii). c) Follows directly from item b); notice that $p = 1$ is the lowest possible value of p, when the consumption of the sin good is not subsidized. ■

Essentially, the consumer prefers more personalized pricing schemes to less personalized ones. One measure of the degree of personalization is the width of the corridor, $p - q \equiv \kappa$. This is a rather natural measure, since in principle the consumer could choose any non-linear pricing system within this corridor. The pure system of sin licenses, which is favored by the consumer, is the most personalized scheme, with corridor width $\kappa = \infty$. Sin taxes are the least personalized scheme, with corridor width $\kappa = 0$.

4.2 Socially optimal pricing scheme

The scheme chosen by the consumer is not socially optimal. In contrast to the consumer's two objectives (minimizing monetary costs as well as distortions), a hypothetical social planner has only one objective, to minimize distortions. For the social planner, one consumer's monetary cost from the pricing scheme (which arises if the consumer price is higher than the producer price, normalized to 1) is another consumer's gain: any tax revenues from the pricing scheme are redistributed back to the consumers. Hence the social planner does not care about the monetary costs of a personalized pricing scheme to any one consumer. (However note that the planner does take into account the producer price, or production cost, of sin goods (normalized to 1), which constitutes a true social opportunity cost.) We can then state the following result.

Proposition 3 *A social planner would never choose a system of sin licenses for any given consumer.*

Proof. See Appendix A 2. ■

We show in Appendix A 2 that whenever the constraints (12) and (13) are not binding, the planner implements the socially optimal consumption scheme $x_i^o(\theta_i)$ (for each consumer i) and the optimal solution would have the marginal price taking the form

$$T_i'(x_i(\theta_i)) = 1 + (1 - \beta_i)\,\delta_i h_i'(x_i(\theta_i)) \tag{14}$$

Hence the marginal price is monotonously increasing in consumption, if the harm function is convex $h''(x) > 0$ (while the marginal price is constant, if harm is linear $h''(x) = 0$). In the case of sin licenses, this is obviously not the case.

We further show in Appendix A 2 that if the costs of the regulatory scheme for any given consumer were neutralized by personalized subsidies, the scheme chosen by the consumer would in fact coincide with the one that the social planner would choose. This further highlights the fact that it is the cost minimization motive that causes the consumer to choose sin licenses rather than the socially optimal scheme.

Note that both cases - the one where the social planner chooses the regulatory mechanism for each consumer, and the one where personalized subsidies are used - should be thought of as hypothetical thought experiments: to implement either of these systems, the social planner would need to have information on each consumer's degree of self-control problems and the harm function associated with sin good consumption.

5 Role of sin licenses in regulating the consumption of harmful goods

Above, we have shown that sin licenses are in general not the socially optimal policy, and (if free to choose) the social planner would never choose a sin licensing scheme for any given consumer. Does this imply that sin licenses have no role to play in regulating harmful consumption? Based on our analysis, such a conclusion would be premature. The first-best policy above is not implementable, as it would require information (e.g. on the extent of individuals' self-control problems) that the social planner is very unlikely to have. The sin licensing scheme, on the other hand, is based on self-selection, and is therefore part of a feasible policy package.

Goods such as alcohol are currently in practice subject to linear taxation - would we do better by replacing linear taxation with a system of sin licenses? The answer to this question is in general ambiguous. Personalized regulation allows for catering to the personalized needs for commitment: this is an argument in favor of sin licenses. However, as was explained above, the cost-minimization objective that the consumers have when choosing among different regulating mechanisms often leads them to prefer a low (zero) tax for low preference shocks. This leads to excessive (laissez-faire) consumption for low preference shocks, and mandatory linear taxes may have some desirable commitment properties compared to sin licenses. Furthermore, in some cases the cost-minimization motive may cause consumers to opt for excessively stringent regulation: for example, if consumers use sin licenses to abstain altogether, tax payments will naturally be zero. After presenting some parametric examples to illustrate these points in Section 5.1, we turn to derive some general results in Sections 5.2 and 5.3.

5.1 Sin taxes vs. sin licenses: numerical examples

Based on Proposition 1 and Corollary 2, we know that, as long as the hazard rate is non-decreasing, $\lambda'(\theta) \geq 0$, an individual consumer prefers being regulated by sin licenses rather than by sin taxes. However, from the social (planner's) point of view, things can sometimes look quite different.

To make this point, we use a simple and deliberately stark example. Assume that the harm function is linear, $h(x) = gx$. Given this assumption, a linear sin tax $\tau = \tau^*(\beta_i) = (1 - \beta_i) g\delta$ and a corresponding linear consumer price $s = s^*(\beta_i) = 1 + (1 - \beta_i) g\delta$ implements the first-best consumption schedule $x^o(\theta)$ for type β_i; that is, the individual consumes the first-best amount with all preference shock realizations θ; see equation (14). Then from the social (planner's) point of view, sin licenses cannot be a better way to regulate type β_i than the linear tax $\tau^*(\beta_i)$. Hence (assuming that $\lambda'(\theta) \geq 0$) here we have one case where the planner's perspective (favoring the sin tax) and the consumer's perspective (favoring sin licenses) starkly differ from each other.

To get a still more concrete feel, we consider a simple parametric example where the utility function $v(x) = x - \frac{1}{2}x^2$, the preference shock is uniformly distributed[7] over $[1, 3]$ and harm function $h(x) = \delta^{-1}x$ so that $\delta h(x) = x$; then the sin tax $\tau^*(\beta) = 1 - \beta$ implements the socially optimal consumption scheme for type β. Table 1 presents the change in the social planner's welfare measure $E_\theta[\Delta W(\theta, \beta)]$ and the individual's welfare measure $E_\theta[\Delta V(\theta, \beta)]$, when there is a transition from a universal linear sin tax τ to pure sin licenses $p = 1, q = \infty$. (We turn to the case of generalized sin licenses below.) We consider three different tax rates $\tau = 0.2$, $\tau = 0.5$, $\tau = 0.9$. and 11 different types β (0, 0.1,....1). In each row, we have marked with an asterisk the individual (β^*) for whom the tax rate implements the socially optimal consumption scheme. For example $\tau = 0.2$ implements socially optimal consumption for $\beta^* = 0.8$.

Table 1 illustrates the difference between the individual's point of view and the social planner's point of view.[8] While the individual's welfare measure rises for all tax rates τ and all types β (panel b of Table 1), for the social welfare measure the pattern is more diverse: the social welfare measure drops for certain types - close to β^* - , and rises for other types - further away from β^* (panel a of Table 1). The aggregate level social welfare implications

[7]Notice that $\lambda(\theta) = \frac{1}{\theta_H - \theta}$ and $\lambda'(\theta) = \left(\frac{1}{\theta_H - \theta}\right)^2 > 0$, where $\theta_H = 3$.

[8]For ease of exposition, we have multiplied all the numbers presented in Table 1, as well as in Figure 3, by 100. Evidently, this is simply rescaling and does not alter the results.

evidently depend on the distribution of types (β) - as well as on the original tax rate: if the majority of the population has types close to β^*, moving from taxes to pure vanilla sin licenses is likely to decrease aggregate social welfare; if a substantial proportion of consumers has types further away from β^*, adopting the system of sin licenses is more likely to improve aggregate social welfare. (More on aggregate welfare implications in Sections 5.2. and 5.3.)

Table 1. Moving from sin tax τ to pure sin licenses $p = 1, q = \infty$

a) Change in the social planner's welfare measure $E_\theta [\Delta W (\theta, \beta; \tau)]$

	Type β										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Tax rate τ											
0.2	8.5	5.3	2.9	1.3	.2	**−.4**	**-.65**	**-.59**	**-.3****	0.0	.3
0.5	.6	**-.8**	**-1.6**	**-2.0**	**-1.9**	**-1.6***	**-1.0**	**-.4**	.4	1.0	1.6
0.9	**-2.9**	**-3.0***	**-2.7**	**-2.1**	**-1.3**	**-.5**	.5	1.3	2.1	2.7	3.0

b) Change in the individual's welfare measure $E_\theta [\Delta V (\theta, \beta; \tau)]$

	Type β										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Tax rate τ											
0.2	15.5	11.5	8.3	5.9	4.1	2.9	2.2	1.7	1.5*	1.5	1.5
0.5	12.1	9.1	6.8	5.1	3.9	3.1*	2.7	2.6	2.6	2.6	2.7
0.9	7.5	5.6*	4.1	3.2	2.7	2.4	2.4	2.6	2.8	3.0	3.0

Specifications: $v(x) = x - \frac{1}{2}x^2,\ h(x) = \delta^{-1}x, \quad \theta \in U[1,3]$
First-best consumption profile: $x^o(\theta) = \max\{0, 1 - 2\theta^{-1}\}$
With taxes: $x_\tau(\theta, \beta; s) = \max\{0, 1 - (s+\beta)\theta^{-1}\},\ s = 1 + \tau$
Welfare measures: planner $W_\tau(\theta, \beta; \tau) = \theta v(x_\tau) - [x_\tau + \delta h(x_\tau)]$
consumer $V_\tau(\theta, \beta; \tau) = \theta v(x_\tau) - [s x_\tau + \delta h(x_\tau)] = W_\tau(\theta, \beta; \tau) - \tau x_\tau$
With licenses: $x_\ell(\theta, \beta) = \max\{0, \min\{1 - (1+\beta)\theta^{-1}, y\}\},\ y = \frac{1}{3}\beta$
Welfare (planner and consumer) $W_\ell(\theta, \beta) = V_\ell(\theta, \beta) = \theta v(x_\ell) - [x_\ell + \delta h(x_\ell)]$
Taxes vs. licenses: $E_\theta[\Delta W(\theta, \beta; \tau)] = E_\theta[W_\ell(\theta, \beta) - W_\tau(\theta, \beta; \tau)]$,
$E_\theta[\Delta V(\theta, \beta; \tau)] = E_\theta[V_\ell(\theta, \beta) - V_\tau(\theta, \beta; \tau)]$
Note: For ease of exposition, all figures in the Table have been multiplied by 100.

By Corollary 2, we also know that each consumer prefers a more personalized pricing scheme, with a wider corridor $q - p = \kappa$ to a less personalized scheme with a narrower corridor. Next we illustrate how the planner's perspective may differ from that of the individual.

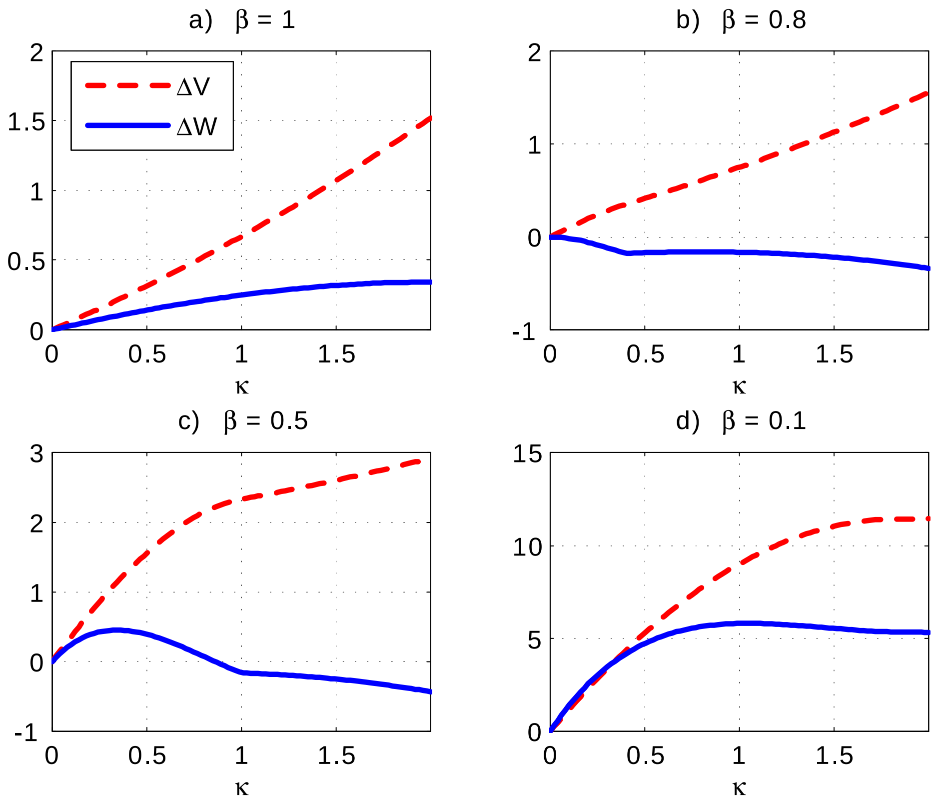

Figure 1: Moving from a universal linear sin tax $\tau = 0.2$, $s = 1 + \tau = 1.2$ to a generalized system of sin licenses with $p = s - \phi\kappa$, $q = s + (1 - \phi)\kappa$, where $\phi = 0.1$ and the width of the corridor $q - p = \kappa$ varies between 0 and 2: change in the private welfare measure $E_\theta [\Delta V (\theta, \beta; s, p, q)]$ and the social welfare measure $E_\theta [\Delta W (\theta, \beta; s, p, q)]$ for four types (β). Consumption under licenses is given by $x_\ell = x_\ell (\theta, \beta; p, q) = \max \{0, \widetilde{x} (\theta, \beta; q), \min \{\widetilde{x} (\theta, \beta; p), y\}\}$, where $\widetilde{x} (\theta, \beta; r) = 1 - (r + \beta) \theta^{-1}$, $r \in \{p, q\}$ and $y = \max \left\{0, \frac{1}{3} \left(2 - \frac{1}{2} (p + q)\right), \frac{1}{3} (\beta - \tau)\right\}$, while the welfare measures under licenses are given by $W_\ell (\theta, \beta; p, q) = \theta v (x_\ell) - 2x_\ell$ and $V_\ell (\theta, \beta; p, q) = \theta v (x_\ell) - (1 + p) x_\ell - \kappa \max \{0, x_\ell - y\} = W (\theta, \beta; q, p) - \tau_1 x_\ell - \kappa \max \{0, x_\ell - y\}$. For other specifications and definitions, see Table 1.

We start out with a sin tax, with a uniform linear consumer price $s = 1 + \tau$, and then construct a system of sin licenses (or rather a sequence of systems) by setting $p = s - \phi\kappa$ and $q = s + (1 - \phi)\kappa$, where $\kappa \geq 0$ measures the width or the corridor (since $q - p = \kappa$) or the degree of personalization, and $\phi \in [0, 1]$. In the numerical example we assume that $\tau = 0.2$[9] so that the original consumer price is $s = 1.2$. We further set $\phi = 0.1$: when the corridor (κ) becomes wider, the ceiling ($q = s + (1 - \phi)\kappa$) moves more than the floor ($p = s - \phi\kappa$). This choice of ϕ is motivated by the fact that the lower bound of the floor, at $p = 1$, is not so far away from the original price $s = 1.2$, while at least in principle the ceiling q has much more room to rise.

Then by letting κ grow from 0 to 2, allows us to consider a sequence of systems, ranging from a uniform linear sin tax at $\kappa = 0$, to what is essentially a system of pure sin licenses at $\kappa = 2$: when $\kappa = 2$ the floor is at $p = 1$ while the ceiling $q = 3$ is so high that in equilibrium no consumer will buy sin goods at price q.[10] When $0 < \kappa < 2$ we then have a sequence of generalized systems of sin licenses, that fall between the two polar cases (see the discussion in Section 4.1, following Corollary 2).

Figure 1 illustrates the welfare consequences of moving from uniform linear sin taxes to a system of (generalized) sin licenses. The figure shows the change in the private welfare measure ($E_\theta [\Delta V (\theta, \beta)]$) and in the social welfare measure ($E_\theta [\Delta W (\theta, \beta)]$) for four consumer types ($\beta = 0.1,\ \beta = 0.5,\ \beta = 0.8,\ \beta = 1$) and different values of $\kappa \in [0, 2]$ (horizontal axis). The figure illustrates that the private welfare measure (red dashed line) rises with the degree of personalization (corridor width κ) for all consumers.This is consistent with Corollary 2.

However, for the social welfare measure ($E_\theta [\Delta W (\theta, \beta)]$), the pattern is more diverse. In our example, the social welfare measure monotonously rises with κ only in the case of the fully rational type $\beta = 1$; see panel a). Meanwhile, for $\beta = 0.8$ (panel b), social welfare $E_\theta [\Delta W (\theta, \beta)]$ actually falls with the corridor width κ. Finally, for types $\beta = 0.5$ and $\beta = 0.1$, the relationship between corridor width κ and social welfare $E_\theta [\Delta W (\theta, \beta)]$ is non-monotonous, and a license system with a relatively narrow corridor κ is beneficial for social welfare.

A further thing to note is that moving from a uniform sin tax to a system of generalized licenses does not necessarily lower the consumer's tax bill: when $q < 3$, in equilibrium the consumer may buy a part of the sin goods at price $q = 1 + \tau_2$, where the tax component $\tau_2 > \tau$. That is, the consumer may be willing to pay a higher price for achieving better self-control than would be possible with a linear tax. If the consumer's tax bill does not go down, the social welfare measure actually improves more than the private welfare measure ($E_\theta [\Delta W (\theta, \beta)] > E_\theta [\Delta V (\theta, \beta)]$). In Figure 1, this is the case for $\beta = 0.1$ with small values

[9] The 20% tax rate is of the same order of magnitude as for example the tax on mild alcohol products in many European countries.

[10] With price q, the demand of sin goods is given by $x(\theta, \beta; q) = \max\{0, 1 - (q + \beta)\theta^{-1}\}$. When $q \geq 3$, $x(\theta, \beta; q) = 0$ even for a myopic consumer ($\beta = 0$) with maximum preference shock $\theta = \overline{\theta} = 3$.

of κ. [11]

Above we considered situations where the price ceiling of the sin license system is higher than the original price with a linear tax (i.e. $q > s = 1 + \tau$), but the price floor is lower (i.e. $p < s$). One might think that any reform that allows consumers to only choose tighter personalized regulation ($p \geq s$, $q > s$) would be guaranteed to improve social welfare (or at least not lower welfare), compared to a linear sin tax τ: under such an arrangement, the consumer cannot (ab)use the system of licenses to acquire sin goods at a lower unit price. However, this conjecture is not necessarily true. The consumer may sometimes choose excessively tight personalized regulation, in order to minimize tax payments. Intuitively, if the system of sin licenses enables the individual to commit to a small amount of sin goods consumption, the individual's monetary costs and tax payments will be small.

This possibility can be illustrated using our example. Let us assume that the prevailing sin tax rate is $\tau = 0.5$; hence the sin tax implements the optimal consumption scheme $\left(x^o(\theta) = \max\left\{0, 1 - 2\theta^{-1}\right\}\right)$ for consumer type $\beta = 0.5$. Next assume that a system of sin licenses is introduced, with $p = s = 1 + \tau = 1.5$, and $q \geq 2.5$ (see footnote 26 below). Hence the consumer can choose personalized regulation that is tighter than the original linear tax. (Choosing a large quota y that is never binding in equilibrium would mean choosing the original system.) One can show that type $\beta = 0.5$ chooses a zero sin license quota, $y = 0$, and in equilibrium he will not consume any sin goods with any preference shock realization.[12] Moving form the optimal consumption scheme $(x^o(\theta))$, under the sin tax $\tau = 0.5$, to zero consumption under sin licenses reduces social welfare by $E_\theta[\Delta W(\theta, \beta)] = -3.05$. However, from the consumer's point of view, the problem with implementing the optimal consumption scheme $x^o(\theta)$ with sin tax $\tau = 0.5$ is that it involves (expected) tax payments $(\tau E_\theta[x^o(\theta)] = 4.7)$. Since committing to zero consumption allows the consumer to avoid the sin tax $(\tau = 0.5)$ altogether, the individual's welfare increases by $E_\theta[\Delta V(\theta, \beta)] = 1.7$.

It is however worth noting that the case $\beta = 0.5$, $\tau = \tau^*(0.5) = 0.5$ was deliberately chosen to make a stark point. It is easy to come up with cases, where more stringent personalized regulation is (socially) beneficial, as it helps people to lower their consumption or to abstain altogether. As a simple illustration consider our parametric example with $\tau = 0.2$ (and $q \geq 3$). Sophisticated types with severe self-control problems, $\beta = 0$, $\beta = 0.1$ and $\beta = 0.2$, stop consuming the sin good altogether with the help of licenses, and this is beneficial both from the consumer's perspective and from the social perspective ($E_\theta[\Delta W(\theta, \beta = 0)] = 8.5$, $E_\theta[\Delta W(\theta, \beta = 0.1)] = 5.2$, $E_\theta[\Delta W(\theta, \beta = 0.2)] = 2.6$).

[11] The pattern, where some consumers' tax bill increases when moving from a linear sin tax to a generalized system of sin licenses, comes out still clearer when the original tax rate is lower, say $\tau = 0.1$.

[12] Remember that $x^*(\theta, \beta; q) = \max\left\{0, 1 - (\beta + q)\theta^{-1}\right\}$. When $\beta = 0.5$ and $q \geq 2.5$ we have $x^*(\theta, \beta; q) = 0$ for all $\theta \in [1, 3]$.)

5.2 Sin taxes vs. sin licenses: more general results

In this section we provide some more general results concerning the social welfare comparison between sin taxes and sin licenses at the aggregate level. We start out with a result that lends some support to sin licenses, or more generally to personalized regulation.

We show in Appendix A 3 that a marginal reform involving (a form of) sin licenses can be socially beneficial under rather mild conditions: supplementing a linear sin tax τ with a system of so called marginal sin licenses would improve social welfare, denoted by W, if $\frac{\partial W}{\partial \tau} \geq 0$ (sufficient condition). By 'marginal sin licenses' we refer to a system where a minimal degree of personalized regulation is introduced on top of a linear tax: compared to a situation where only a linear tax τ is in place, the consumer chooses a level of consumption y above which a marginally higher tax rate $\tau + d\tau$ is applied. The intuition for the condition $\frac{\partial W}{\partial \tau} \geq 0$ is the following. In this case a small tax increase would improve aggregate social welfare - or at least not lower welfare - but introducing a marginal sin license instead has the added benefit of allowing for self-selection: the tax increase is implemented for those consumers for whom tighter regulation is beneficial. This result also implies that introducing a marginal sin license will improve welfare compared to the *optimal* linear tax.[13]

Hence a universal linear sin tax is in general not the best feasible regulatory system for sin goods. On the other hand, the society should not adopt pure sin licenses either, even if this option were in principle feasible (i.e. the system were not undermined by potential secondary market trade). We show in Appendix A 3 also that supplementing a system of pure sin licenses (where $\tau_1 = 0$ and $p = 1$) with a (marginal) linear sin tax ($\tau_1 > 0$) would improve welfare. The positive tax rate τ_1 is useful as it imposes some commitment in cases where a pure system of sin licenses is excessively geared towards flexibility (at low levels of the preference shock). These results are summarized in the following proposition:

Proposition 4 *(i) Supplementing a system of sin licenses with a (marginal) linear sin tax would improve welfare. (ii) Supplementing a linear sin tax τ with a system of (marginal) sin licenses would improve social welfare, if $\frac{\partial W}{\partial \tau} \geq 0$ (sufficient condition).*

Proof. See Appendix A3. ■

Taken together, the arguments presented above indicate that linear sin taxes and sin licenses can be thought of as complements, not substitutes: it is not optimal to rely on one type of regulation - linear tax or pure sin licenses - only, but social welfare can be improved if elements of one system are introduced to supplement the other. To put it differently, some form of generalized sin licenses is superior to both linear sin taxes and pure sin licenses.

[13] Haavio and Kotakorpi (2011) analyze the political economy of sin taxes.They show that equilibrium sin taxes are likely to be below the social optimum and the condition $\frac{\partial W}{\partial \tau} \geq 0$ would then hold.

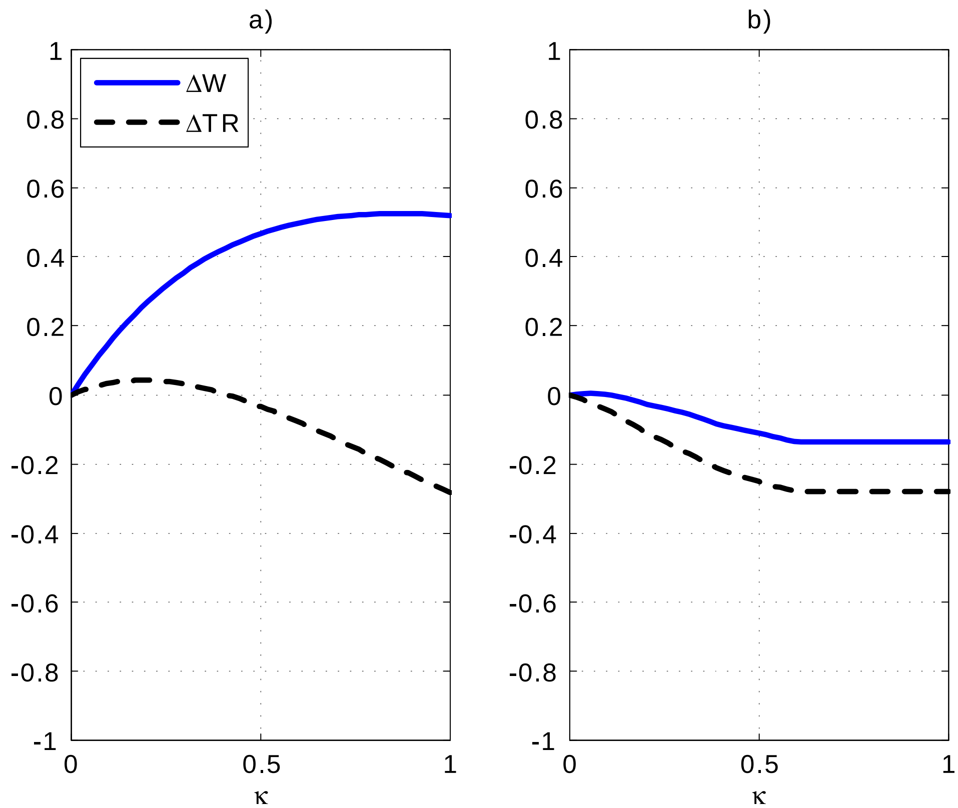

Figure 2: Moving from a universal linear sin tax τ, $s = 1 + \tau$ to a generalized system of sin licenses where $p = s$ and $q = s + \kappa$, and the corridor width κ varies over $[0, 1]$ (horizontal axis). Change in the aggregate social welfare measure $E_{\theta,\beta}\left[\Delta W\left(\theta, \beta; s, p, q\right)\right]$, and aggregate tax revenues $E_{\theta,\beta}\left[\Delta TR\left(\theta, \beta; s, p, q\right)\right]$. Panel a) has 11 types β (0,0.1,0.2,...,1): we assume that 75% of the consumers are fully rational ($\beta = 1$), while the remaining "irrational" types each have an equal frequency mass 2.5%. Panel b) has 3 types β with a positive mass (0.7,0.8,0.9): we assume that 51% of the consumers are of type $\beta = 0.9$, while the remaining types ($\beta = 0.7$ $\beta = 0.8$) each have an equal frequency mass 24.5%. In both panels (a and b) the original tax rate $\tau = \tau^{**} = 0.17$; in both cases the same tax rate $\tau^{**} = 0.17$ is chosen in a (majority voting) political equilibrium. The equilibrium tax rate τ^{**} is computed using equation (13) in Haavio and Kotakorpi (2011). Also note that in case a), the social welfare maximizing tax rate τ^{o} =0.26, while in case b) τ^{o} =0.18. For other specifications and definitions, see Table 1 and Figure 1.

However, the results presented in Proposition 4 concern marginal reforms, and therefore do not tell us whether any given reform is welfare improving. Moreover, marginal reforms arguably bring about marginal welfare improvements, which may well be outweighed by administrative costs. The following Proposition provides a (sufficient) condition that allows one to assess ex-post whether a given (non-marginal) reform is welfare improving. The proposition concerns replacing a linear sin tax with a system of generalized sin licenses.

Proposition 5 *Assume that consumers are sophisticated and that the hazard rate condition holds, $\lambda'(\theta) \geq 0$. Suppose that a linear sin tax τ is replaced with a generalized system of sin licenses where $\tau_1 \leq \tau \leq \tau_2$. This reform would improve aggregate social welfare if it does not lead to a reduction in tax revenue (sufficient condition).*

Proof. See Appendix A4. ■

Furthermore, the hazard rate condition ($\lambda'(\theta) \geq 0$) can be dropped, if each consumer can choose between staying in the tax regime and switching to sin licenses. In this situation, a consumer switches if and only if he privately benefits from the change. When $\lambda'(\theta) \geq 0$ for all consumers, Proposition 1 implies that sophisticated individuals would anyway opt for sin licenses, rather than the linear tax τ

The intuition for Proposition 5 is that if tax revenue does not decline after the introduction of sin licenses, we can infer that the consumers' choices of license quotas were (on the whole) not determined by the motivation to minimize costs (i.e. to escape the sin tax), but rather by the motivation to minimize distortions; the latter is the motivation that the social planner would agree with, and hence sin licenses are guaranteed to improve welfare. A nice feature of the result is that the implied criterion only involves a measure which is readily observable - the change in (aggregate) tax revenue. Note that the result is *not* related to any concern for tax revenue per se. The sole objective for the social planner is to minimize distortions related to self-control problems in sin goods consumption.[14]

The tax revenue criterion is a sufficient condition, and sin licenses may improve social welfare even if the criterion is not met. For an illustration see Figure 2. The Figure shows the change in tax revenue and welfare when moving from a universal linear sin tax to a system of generalized sin licenses. The corridor width $\kappa = q - p$ is a measure of the degree of personalization associated with the sin licensing scheme. In the situation depicted in panel a), the change in tax revenue signals a positive outcome for reforms involving a license system with corridor width $\kappa \in [0, 0.4]$ (and tax rate τ_2 between 17% and roughly 60%), while sin licenses actually improve social welfare with any $\kappa > 0$.[15] On the other hand, using the tax

[14] This is a reasonable objective. The conclusion from the literature on optimal commodity taxation is that as a general rule, from the point of view of efficient revenue collection, commodity taxes should in practice be uniform across goods - see e.g. Crawford, Keen and Smith (2010). Hence the main (legitimate) reason for higher taxes on sin goods is to eliminate distortions.

[15] Attempts to implement a system of licenses with a wide corridor ($\kappa > 0.4$) could be arguably undermined by (potential) secondary market trade. Hence the failure of the tax revenue criterion to signal a positive outcome for high κ might not be such a big problem.

revenue criterion rules out the risk of a false positive, i.e. signaling a positive welfare outcome from a reform, when there is none. This is illustrated in Panel (b) of Figure 2: Here the reform is roughly welfare neutral (or only just positive) for sin licenses with a narrow corridor κ, and actually lowers social welfare for a wider corridor κ. Panel (b) shows that in this case tax revenues fall as a result of the reform.

Furthermore, we show in online Appendix B 6 that sin licenses are more likely to improve welfare, compared to sin taxes, if consumers differ significantly from each other in terms of self-control problems. This is intuitive: If consumers are more or less identical, a uniform linear sin tax provides a good fix to self-control problems.. If the consumers differ in their need for commitment, there is more room for personalized regulation. The finding is also illustrated in Figure 2, where the distribution of self-control problems is more dispersed in panel a) where $std(\beta) = 0.28$ than in panel b) where $std(\beta) = 0.083$.

Finally note that in the cases illustrated in Figure 2, we assume that consumers can only adopt a system of stricter personalized regulation, on top of the pre-existing sin tax. A key reason for this focus is the fact that under such an arrangement, the tax revenue criterion is robust even when there are naive consumers in the economy. Next we turn to the question of naivete.

5.3 Naivete

So far, we have assumed that consumers are sophisticated, i.e. they are fully aware of their degree of self-control problems. Naive individuals, on the other hand, are unaware of their self-control problem. We need to somewhat refine the policy implications of our analysis, if there are naive consumers in the economy. Assume that the society moves from a uniform linear sin tax τ to a system of sin licenses (τ_1, τ_2), such that $\tau_1 < \tau < \tau_2$. Importantly, under the new regime, a naif chooses the lightest possible regulation, even when this choice is detrimental for him from a private perspective. Note that true private welfare depends on i) monetary costs, ii) flexibility in the face of preference shocks and iii) commitment. The naive individual only takes into account items i) and ii): as the naif thinks that he will consume optimally in the future, he sees no value in commitment. Quite intuitively, the case where a naive individual's choice of sin licenses reduces his welfare is most likely, if the naive person has severe self-control problems (β is close to 0) and would hence benefit from commitment through a higher tax rate.

This finding has problematic implications for our reform evaluation criterion (see Proposition 5). If there are naive consumers in the economy, a non-negative change in tax revenue does not guarantee that the introduction of personalized regulation has improved social welfare, as the naive individual does not take into account the corrective effect of the regulating scheme in his choice of sin licenses.

How can we design a personalized regulation scheme and a reform evaluation criterion that are robust even when some of the consumers are naive? There is a relatively simple answer to this question: this can be done by supplementing the current linear tax with a system of sin licenses, where the individual is not allowed to opt out from the linear tax, but can implement stricter personalized regulation on top of the tax. More formally, the system of sin licenses involves tax rates $\tau_1 = \tau$ and τ_2. This reform would have no effects on naives (who would simply choose the original tax rate τ), whereas it would, under certain conditions, improve welfare for sophisticates. Hence, we have a corollary of Proposition 5.

Corollary 6 *Assume that there are both sophisticated and naive consumers in the economy. Suppose that a linear sin tax τ is supplemented with a system of sin licenses, $\tau_1 = \tau$ and $\tau_2 > \tau$, so that an individual consumer can implement stricter personalized control on top of the linear tax. This reform would improve aggregate social welfare if it does not lead to a reduction in tax revenue (sufficient condition).*

Notice that we do not need to impose the hazard rate condition (i.e. the requirement that $\lambda'(\theta) \geq 0$) here: if a (sophisticated) consumer does not (privately) benefit from the licenses, he just sticks to the linear tax τ.

6 Discussion and policy implications

Evidence on personalized regulation of harmful consumption is hard to find, as real tax schedules tend to be linear. An interesting piece of evidence - albeit highly suggestive - comes from alcohol control policies in Finland and Sweden mainly in the 1940s and 1950s.[16] In both countries, official purchases of alcohol were only allowed upon showing a special identity card or permit, and purchases were recorded on the card. The systems had elements of personalized regulation: for example, the permit could be denied from individuals suspected of misuse. (Häikiö 2007.) These systems differ from sin licenses in that they were not voluntary, as it was not possible to opt out from the scheme. Nevertheless, these examples may provide some evidence on the self-control properties of personalized regulation, as individuals had the possibility to opt for *stricter* regulation by choosing not to obtain a permit.

There is some suggestive evidence that for some individuals, not obtaining a permit may have functioned as a self-control device. During the latter part of the 1940s, when the Finnish system was in full operation, only 30-50% of individuals over the age 20 obtained a permit each year. (Häikiö 2007.) At the same time, abstention rates were relatively high: in 1946,

[16] In 2012, some Dutch cities implemented a system of "weed passes", whereby residents were required to register for a pass to be allowed entrance to coffee shops. This system has some similarities with sin licenses even though the main aim was to reduce drug-related tourism. There were some reports of reduced drug tourism and increased street-dealing, as well as some residents being hesitant to register for a pass. (See e.g. http://www.observantonline.nl/English/Home/Articles/tabid/128/articleType/ArticleView/articleId/2506/The-sticky-matter-of-marijuana-in-Maastricht.aspx.) We are not aware of any systematic discussion of whether the system might have helped some residents to reduce their cannabis consumption.

20 % of Finns had never had any alcohol (a very strict definition of abstinence) and 49 % had not drunk alcohol within the past month. (Sulkunen 1979.).

On the other hand, widespread secondary markets posed challenges for these systems. For instance in Sweden, a large proportion of offences associated with the misuse of alcohol were committed by individuals who did not hold a permit. In Finland, a significant proportion of individuals whose permit was withdrawn for a fixed period never re-applied for it. Many of these individuals however had not stopped alcohol consumption, but rather found unofficial means of obtaining alcohol more convenient than the official route (Immonen 1980).

This evidence is broadly consistent with our story: personal licenses may have helped some individuals to limit their alcohol consumption, but unofficial trade caused problems for the system. The prevalence of unofficial trade also suggests that indeed a strict limit on consumption may not be optimal, but the type of generalized sin licenses that we consider - where the tax rate is higher but less than infinite above a certain cut-off level of consumption - may be a more promising alternative.

Personalized regulation is rarely seen in practice, but might such schemes be worth considering for policy purposes? Sometimes schemes that first may appear to be theoretical curiosities - consider for example the case of tradeable emission permits - are implemented in the real world. A first contribution of our analysis is to narrow down the range of personalized schemes that one might consider. There are an infinite number of possible personalized schemes, and they are indeed potentially very complex. We show that if sophisticated consumers are allowed to select any general, linear or non-linear personalized pricing scheme, they will in fact choose a relatively simple scheme of sin licenses. In this sense, our model predicts that a system of sin licenses would emerge as an equilibrium outcome, in case personalized regulation were adopted.

We have studied the conditions under which sin licenses can improve social welfare compared to a linear sin tax. This comparison depends on several factors, notably (i) the distribution of self-control problems in the economy; (iii) existence of secondary markets; (iii) presence of naive consumers; and (iv) administration costs. We also derived a simple condition to assess whether introducing sin licenses improves social welfare.

First, we have shown that sin licenses are more likely to produce welfare gains, if people differ significantly in terms of the severity of self control problems. Quite intuitively, in such a situation there is more need for personalized regulation. Second, secondary market trade constrains the space for personalized regulation: the lower the costs of secondary market trade, the less the personalized pricing schemes can differ from universal linear pricing. This may lower the potential welfare gains from sin licenses, but not necessarily so. While we have shown that an individual consumer would want to have the lowest possible price floor (for purchases with sin licenses) and the highest possible price ceiling (for purchases without a license) to acheive the (privately) best combination of low monetary costs and credible commitment, this view is not typically shared by the social planner. From the social point of view, a higher price floor is preferable, as it improves commitment, and often also a lower price ceiling is preferable, as it allows some flexibility. Hence, the possibility of secondary market trade, which constrains the price floor and the price ceiling from being too far apart, does not necessarily constitute an insurmountable hurdle for personalized regulation.

Third, the potential for personalized regulation appears to be compromised by the presence of naive consumers: Naive individuals, who only consider low costs and high flexibility while ignoring the need for commitment, always choose the lightest possible form of regulation, even when this were not in their (true) self interest.

However, our analysis suggests a reform that may work when there are both sophisticated and naive consumers: such a reform involves allowing consumers to supplement the universal linear tax with *stricter* personalized regulation. That is, consumers would be subject to the regular linear sin tax, but could opt for tighter regulation by adopting a personalized ceiling on consumption above which a higher tax rate applies. The aim of this reform is to target sophisticated consumers only, while naive consumers stick to the original linear tax.

Our analysis provides a simple criterion that can be used to assess whether the introduction of sin licenses is welfare improving. This is the case if the reform does not lead to a reduction of tax revenues (sufficient condition). If this criterion is met, we can infer that the consumers' choices of license quotas were on the whole not determined by the motivation to minimize costs (to the individual himself), but rather by the motivation to minimize distortions. The tax revenue criterion can be applied to any system of sin licenses, if all consumers are sophisticated. If there are naives in the economy, the tax revenue criterion is robust as long as the choice of sin licenses is limited to adopting stricter personalized regulation compared to the existing linear scheme.

Finally, administrative costs can be expected to be higher in the case of personalized regulation than for linear schemes. Cowell (2008) discusses the possibilities for using smart cards to implement personalized taxation. The tax revenue condition referred to above is a sufficient condition for sin licenses to improve welfare, if administrative costs are not taken into account. It also follows that if the reform increases tax revenue by an amount that exceeds the additional administrative costs, the reform is welfare improving. One should remember that this result has nothing to do with a concern for tax revenue or the fiscal positition of the government per se. In our view it is interesting and useful that the desirability of the reform can nevertheless be assessed by such a simple fiscal criterion. In a nutshell, if the reform is self-financing, it is also guaranteed to be welfare improving.

Appendix A: Proofs of propositions

A1. Proof of Proposition 1

Consider the mechanism design problem characterized by (9)-(13). From the ex post self's problem (11) we get the first-order incentive constraint

$$\frac{\partial \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}} = 0 \tag{15}$$

or, using (11)

$$\left[\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) - T'(x(\theta))\right]\frac{dx}{d\theta} = 0. \tag{16}$$

We also have the second-order incentive constraint

$$\frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}^2} \leq 0. \tag{17}$$

By totally differentiating (15) we get

$$\frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}^2} + \frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}\partial\theta} = 0 \Leftrightarrow \frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}^2} = -\frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}\partial\theta}. \tag{18}$$

Combining (17) and (18) shows that the second-order incentive constraint can be also expressed as $\frac{\partial^2 \widehat{V}(\theta,\theta)}{\partial \widehat{\theta}\partial\theta} \geq 0 \Leftrightarrow v'(x(\theta))\frac{dx}{d\theta} \geq 0$ and finally, since $v'(x(\theta)) > 0$, the second-order condition boils down to

$$\frac{dx(\theta)}{d\theta} \geq 0. \tag{19}$$

That is, consumption must be non-decreasing in θ.

We also assume that the pricing scheme has to satisfy constraints (12) and (13). To sum up, the ex ante self maximizes (9) subject to (16), (19), (12) and (13).

To solve the problem, we first eliminate $T(x(\theta))$ from the ex ante self's objective. To do so, let us define

$$\widehat{w}(\theta) = \widehat{V}(\theta,\theta). \tag{20}$$

Then, the first-order incentive condition (16) implies that $\frac{d\widehat{w}(\theta)}{d\theta} = \frac{\partial \widehat{V}(\theta,\theta)}{\partial\theta} = v(x(\theta))$ (this is just the envelope theorem) and hence

$$\widehat{w}(\theta) = \widehat{w}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta}. \tag{21}$$

Then using (11), (20) and (21) gives

$$T(x(\theta)) = \theta v(x(\theta)) - \beta\delta h(x(\theta)) - \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - \widehat{w}(\underline{\theta}). \tag{22}$$

Next, plugging (22) into (10) yields

$$V(\theta) = \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - (1-\beta)\,\delta h\,(x(\theta)) + \widehat{w}(\underline{\theta})$$

and expected utility is given by

$$E_\theta\,[V(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - (1-\beta)\,\delta h\,(x(\theta)) + \widehat{w}(\underline{\theta})\right] f(\theta)\,d\theta \tag{23}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[v\,(x(\theta))\,\frac{1}{\lambda(\theta)} - (1-\beta)\,\delta h\,(x(\theta))\right] f(\theta)\,d\theta + \widehat{w}(\underline{\theta}) \tag{24}$$

where $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is the hazard rate and

$$\widehat{w}(\underline{\theta}) = \underline{\theta} v\,(x(\underline{\theta})) - \beta\delta h\,(x(\underline{\theta})) - T\,(x(\underline{\theta})) + B + \Pi \tag{25}$$

is the utility of the *ex post* self with the lowest possible preference shock realization $\underline{\theta}$. From (24) and (25), the (privately) optimal pricing scheme involves minimizing payments $T\,(x(\underline{\theta}))$. Given (12), this implies $T'(x) = p$ for all $x < x(\underline{\theta})$; hence we get $T\,(x(\underline{\theta})) = px\,(\underline{\theta})$. Then the ex ante consumer maximizes

$$J = \int_{\underline{\theta}}^{\overline{\theta}} \left[v\,(x(\theta))\,\frac{1}{\lambda(\theta)} - (1-\beta)\,\delta h\,(x(\theta))\right] f(\theta)\,d\theta + \phi\,(x(\underline{\theta}), \underline{\theta}) \tag{26}$$

where

$$\phi\,(x(\underline{\theta}), \underline{\theta}) = \underline{\theta} v\,(x(\underline{\theta})) - \beta\delta h\,(x(\underline{\theta})) - px\,(\underline{\theta}), \tag{27}$$

subject to (19), (12) and (13).

Next we turn to the pricing constraints (12) and (13). The incentive constraint (16) implies $T'(x(\theta)) = \theta v'(x(\theta)) - \beta\delta h'(x(\theta))$. But then the pricing constraint (12) can be re-expressed as

$$x(\theta) \leq x^*(\theta, \beta; p) \tag{28}$$

while the pricing constraint (13) can be re-expressed as

$$x(\theta) \geq x^*(\theta, \beta; q)\,. \tag{29}$$

where $x^*(\theta, \beta; p)$ and $x^*(\theta, \beta; q)$ are given by (3), with $s = p$ and $s = q$, respectively. Clearly, since $x^*(\theta, \beta; p) > x^*(\theta, \beta; q)$ at most one of the constraints (28) or (29) can bind for each θ. Next, if either of the constraints (28) or (29) holds as an equality, the second-order incentive constraint (19) is automatically satisfied:

$$\frac{dx}{d\theta} = \frac{v'(x^*(\theta, \beta; s))}{\beta\delta h''(x^*(\theta, \beta; s)) - \theta v''(x^*(\theta, \beta; s))} > 0, \;\; s \in \{p, q\} \tag{30}$$

where (30) follows from totally differenciating (3). Hence we can conclude that for each θ, at most one of the constraints (19), (28) and (29) can bind.

Maximization problem.

We want to maximize (26) subject to (19), (28) and (29). We conduct our analysis in four steps. In the first step, we solve the problem (26) disregarding the constraints, and get an unconstrained candidate solution $(x^u(\theta))$. In the second step, we show that the unconstrained solution can be implemented with sin licenses only in the special case, where the hazard rate $\lambda(\theta)$ is constant. In the third step, we establish conditions under which the unconstrained solution $(x^u(\theta))$ does *not* satisfy the second-order incentive constraint (19). In the fourth step we show that if the unconstrained solution $(x^u(\theta))$ does *not* satisfy the second-order incentive constraint (19), the allocation $x(\theta)$ chosen by the consumer *can* be implemented with sin licenses.

Step 1: Unconstrained candidate solution

We maximize (26), taking $x(\theta)$ as the (sequence of) control variable(s).The first-order conditions characterizing the candidate solution $x^u(\theta)$ are of the form

$$v'(x^u(\theta))\frac{1}{\lambda(\theta)} - (1-\beta)\,\delta h'(x^u(\theta)) = 0 \text{ for } \theta \in (\underline{\theta}, \overline{\theta}] \tag{31}$$

and

$$\underline{\theta}v'(x^u(\underline{\theta})) - \beta\delta h'(x^u(\underline{\theta})) - p = 0 \tag{32}$$

The expression (31) gives our unconstrained candidate solution $x^u(\theta)$ for $\theta \in (\underline{\theta}, \overline{\theta}]$. (Since (26) is concave in $x(\theta)$, the second-order condition is satisfied.) The first-order condition (32) implies that at the lowest possible preference shock realization $\underline{\theta}$, we have $x^u(\underline{\theta}) = x^*(\underline{\theta}, \beta; p)$.

Step 2: The unconstrained solution cannot be typically implemented with sin licenses.

In the special case where $\lambda(\theta) = \lambda$ is constant for all θ, unconstrained consumption $x^u(\theta)$ is also constant for all $\theta \in (\underline{\theta}, \overline{\theta}]$; see equation (31). This allocation *can* be implemented with (generalized) sin licenses, with the license quota y given by

$$v'(y)\frac{1}{\lambda} - (1-\beta)\,\delta h'(y) = 0 \tag{33}$$

Otherwise (when $\lambda(\theta)$ is not constant), however, $x^u(\theta)$ varies with shock realization θ. Then combining the first-order condition (31) with the incentive constraint (16), we can see that

$$T'(x^u(\theta)) = \theta v'(x^u(\theta)) - \beta\delta h'(x^u(\theta)) = [(1-\beta)\,\theta\lambda(\theta) - \beta]\,\delta h'(x^u(\theta)), \;\; \theta \in (\underline{\theta}, \overline{\theta}] \tag{34}$$

is the slope of the pricing scheme that implements the unconstrained solution $x^u(\theta)$. Clearly, the unconstrained solution *cannot* be implemented with a system of sin licenses, but a more general non-linear personalized pricing scheme is needed.

Step 3: When is the unconstrained solution (not) a part of an optimal pricing scheme?

Since the unconstrained solution $x^u(\theta)$ can be implemented with sin licenses only in the special case $\lambda(\theta) = \lambda =$ constant, we next try to look for situations where $x^u(\theta)$ is *not* a part of the optimal incentive compatible mechanism. To be more specific, we check whether or not $x^u(\theta)$ satisfies the second-order incentive constraint (19).

From (31) we get

$$\frac{v'(x(\theta))}{(1-\beta)\,\delta h'(x(\theta))} = \lambda(\theta), \theta \in (\underline{\theta}, \overline{\theta}] \tag{35}$$

and differentiating (35) gives

$$\left(\frac{(1-\beta)\,\delta h'(x(\theta))\,v''(x(\theta)) - v'(x(\theta))\,(1-\beta)\,\delta h''(x(\theta))}{[(1-\beta)\,\delta h'(x(\theta))]^2}\right)\frac{dx(\theta)}{d\theta} = \lambda'(\theta) \tag{36}$$

From (36) one can see that

$$\frac{dx(\theta)}{d\theta} \geq 0 \text{ iff } \lambda'(\theta) \leq 0, \quad \frac{dx(\theta)}{d\theta} < 0, \text{ iff } \lambda'(\theta) > 0, \; \theta \in (\underline{\theta}, \overline{\theta}] \tag{37}$$

In words, the unconstrained solution $x^u(\theta)$ satisfies the second-order incentive constraint (19), if the hazard rate $\lambda(\theta)$ is non-increasing in θ, while the unconstrained solution does not satisfy the second-order incentive constraint (19), if the hazard rate $\lambda(\theta)$ is increasing in θ.

While the condition (37) applies for $\theta \in (\underline{\theta}, \overline{\theta}]$, we still need to study the lower bound $\underline{\theta}$. The second-order incentive constraint (19) is satisfied if $x^u\left(\underline{\theta}^+\right) \equiv \lim_{\theta \to \underline{\theta}^+} x^u(\theta) \geq x^u(\underline{\theta}) = x^*(\underline{\theta}, \beta; p)$, while it is *not* satisfied if $x^u\left(\underline{\theta}^+\right) < x^u(\underline{\theta})$, where $x^u\left(\underline{\theta}^+\right)$ is given by (31) and $x^u(\underline{\theta})$ is given by (32). Using (31) and (32) one can show that

$$\begin{aligned} x^u\left(\underline{\theta}^+\right) &\geq x^u(\underline{\theta}), \text{ iff } [(1-\beta)\,\underline{\theta}\lambda(\underline{\theta}) - \beta]\,\delta h'(x^*(\underline{\theta}, \beta; p)) \leq p \\ x^u\left(\underline{\theta}^+\right) &< x^u(\underline{\theta}), \text{ iff } [(1-\beta)\,\underline{\theta}\lambda(\underline{\theta}) - \beta]\,\delta h'(x^*(\underline{\theta}, \beta; p)) > p \end{aligned} \tag{38}$$

Clearly, it is more likely that the second-order incentive constraint is *not* satisfied at $\underline{\theta}$, if the price floor p is low and/or the consumer has severe self-control problems.

Step 4: Optimal scheme when $\lambda'(\theta) > 0$: sin licenses

If $\lambda'(\theta) > 0$, condition (37) is not satisfied. Then (31) is not a valid solution, and we have to take explicitly into account the second-order incentive constraint $\frac{dx(\theta)}{d\theta} \geq 0$. Next, we show that under these circumstances the consumer's preferred allocation and pricing scheme can be implemented with sin licenses.

In what follows we focus on the case where $\lambda'(\theta) > 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. In this situation it is clear that the solution $x(\theta)$ is such that one of the constraints (19), (28) or (29) binds at each point $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. In Appendix B 3 we show that the solution $x(\theta)$ may be implementable with sin licenses even when $\lambda'(\theta) > 0$ holds only in a subset of $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, while in the complementary subset $\lambda'(\theta) \leq 0$. In Appendix B 3 we further show that, if $x^u\left(\underline{\theta}^+\right) < x^u(\underline{\theta})$ (so that the unconstrained solution does not satisfy the second-order incentive constraint at the lower bound $\underline{\theta}$), the solution may be implementable with sin licenses even when $\lambda'(\theta) > 0$ for all θ. (The case where $\lambda'(\theta) > 0$ for all θ is possible if the support of the distribution of θ has no finite upper bound, and $\overline{\theta} = \infty$.)

We now proceed as follows. We adopt $g(\theta) = \frac{dx(\theta)}{d\theta}$ as the control variable, while $x(\theta)$ is the state variable. We then use standard methods of optimal control. The *Lagrangian* associated with the problem takes the form

$$\mathcal{L} = H + \eta_1(\theta)\left[x^*(\theta, \beta; p) - x(\theta)\right] - \eta_2(\theta)\left[x^*(\theta, \beta; q) - x(\theta)\right] \tag{39}$$

where H is the *Hamiltonian*

$$H = \left[v(x(\theta))\frac{1}{\lambda(\theta)} - (1-\beta)\delta h(x(\theta))\right] f(\theta) + \mu(\theta) g(\theta)$$

and $\mu(\theta)$, $\eta_1(\theta)$ and $\eta_2(\theta)$ are Lagrange multipliers associated with the constraints (19), (28), and (29), respectively. Also remember that at each point θ, only one of the constraints (19), (28), and (29) holds. Finally, we need to take into account the salvage term $\phi\left(x(\underline{\theta}), \underline{\theta}\right)$ appearing in the objective function (26).

The optimality conditions from this exercise are the following:

The first-order condition with respect to the control variable g takes the form

$$\begin{array}{c} \mu(\theta) = 0 \text{ if } g(\theta) > 0 \\ \mu(\theta) < 0 \text{ if } g(\theta) = 0 \end{array} \tag{40}$$

The law of motion of the Lagrangian multiplier $\mu(\theta)$ is

$$\frac{d\mu(\theta)}{d\theta} = -H_x = \left[(1-\beta)\delta h'(x(\theta)) - v'(x(\theta))\frac{1}{\lambda(\theta)}\right] f(\theta), \text{ if } g(\theta) = 0 \tag{41}$$

Since the Lagrangian multiplier $\mu(\theta)$ is the marginal value of the state variable $(x(\theta))$, at the lower bound $\underline{\theta}$, the multiplier $\mu(\underline{\theta})$ must be equal to to the marginal contribution of the state to the salvage term $\phi\left(x(\underline{\theta}), \underline{\theta}\right)$ (multiplied by -1, as the salvage term is at the lower bound)

$$\mu(\underline{\theta}) = -\frac{\partial\phi\left(x(\underline{\theta}), \underline{\theta}\right)}{\partial x} = -\left[\underline{\theta}v'(x(\underline{\theta})) - \beta\delta h'(x(\underline{\theta})) - p\right] \tag{42}$$

The Kuhn-Tucker condition associated with the constraint (28) is

$$\begin{array}{c} \eta_1(\theta) = 0 \text{ if } x(\theta) < x^*(\theta, \beta; p) \\ \eta_1(\theta) > 0 \text{ if } x(\theta) = x^*(\theta, \beta; p) \end{array} \tag{43}$$

The Kuhn-Tucker condition associated with the constraint (29) is

$$\begin{aligned}\eta_2(\theta) = 0 \text{ if } x(\theta) > x^*(\theta,\beta;q) \\ \eta_2(\theta) > 0 \text{ if } x(\theta) = x^*(\theta,\beta;q)\end{aligned} \tag{44}$$

Using (40), (41), (42), (43) and (44), solution boils down to

$$x(\theta) = \begin{cases} x^*(\theta,\beta;p) & \text{for } \theta < \theta_1 \\ y & \text{for } \theta_1 \leq \theta \leq \theta_2 \\ x^*(\theta,\beta;q) & \text{for } \theta > \theta_2 \end{cases}$$

where y, θ_1 and θ_2 are determined in the following way: First assume that $\theta_1 > \underline{\theta}$. Then $x(\underline{\theta}) = x^*(\underline{\theta},\beta;p)$ and $\mu(\underline{\theta}) = -\frac{\partial\phi(x^*(\underline{\theta},\beta;p),\underline{\theta})}{\partial x} = 0$. Since the first-order conditions (40) imply that $\mu(\theta_1) = \mu(\theta_2) = 0$ we must have $\int_{\theta_1}^{\theta_2} \frac{d\mu(\theta)}{d\theta} d\theta = 0$. Then using (41), we get the condition

$$\int_{\theta_1}^{\theta_2} \left[(1-\beta)\,\delta h'(y) - v'(y)\frac{1}{\lambda(\theta)}\right] f(\theta)\,d\theta = 0 \tag{45}$$

Alternatively if $\theta_1 \leq \underline{\theta}$, we have $x(\underline{\theta}) = y < x^*(\underline{\theta},\beta;p)$ and $\mu(\underline{\theta}) = -\frac{\partial\phi(y,\underline{\theta})}{\partial x} < 0$. Since $\mu(\theta_2) = 0$, by (40), we get $\mu(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_2} \frac{d\mu(\theta)}{d\theta} d\theta = 0$; then using (41) and (42) we get the condition

$$\int_{\underline{\theta}}^{\theta_2} \left[(1-\beta)\,\delta h'(y) - v'(y)\frac{1}{\lambda(\theta)}\right] f(\theta)\,d\theta = \underline{\theta}v'(y) - \beta\delta h'(y) - p \tag{46}$$

In Appendix B 3, we further show that the conditions (45) and (46) can be both re-expressed in the form (8). At the lower boundary θ_1, the constraint in (43) holds as an equality, yielding (6), while at the upper boundary θ_2 the constraint in (44) holds as an equality, yielding (7). To sum up, θ_1, θ_2 and y are determined by (6), (7) and (8). Hence these results are identical to the ones we derived in our earlier analysis on sin licenses.

Above we have studied interior solutions, such that $y \in (0, x^{\max})$, where $x^{\max} = x^*\left(\overline{\theta},\beta;p\right)$. However, the ex ante consumer's optimization problem may also have a *corner solution.*

i) *Maximum flexibility, minimum cost.* The ex ante consumer's optimal solution is such that the constraint (28) binds for all $\theta \in \left[\underline{\theta},\overline{\theta}\right]$. The consumer can implement this maximum-flexibility-minimum-cost solution with sin licenses, by choosing a large quota $y \geq x^*\left(\overline{\theta},\beta;p\right)$ which always allows him to buy his entire consumption at the minimum price, p. Essentially this then means that the ex ante consumer chooses to implement maximum feasible consumption $x^*(\theta,\beta;p)$ for all realizations $\theta \in \left[\underline{\theta},\overline{\theta}\right]$.

This solution is chosen by fully rational consumers $(\beta = 1)$. To see this notice, that when $\beta = 1$, the consumer's objective function boils down to

$$J = \int_{\underline{\theta}}^{\overline{\theta}} \left[v(x(\theta))\frac{1}{\lambda(\theta)}\right] f(\theta)\,d\theta + \phi(x(\underline{\theta}),\underline{\theta})$$

Here the integral $\int_{\underline{\theta}}^{\overline{\theta}} \left[v\left(x\left(\theta\right)\right) \frac{1}{\lambda(\theta)} \right] f\left(\theta\right) d\theta$ is maximized by making $x\left(\theta\right)$ as large as possible for each $\theta \in (\underline{\theta}, \overline{\theta}]$. But $x\left(\theta\right) = x^*\left(\theta, \beta; p\right)$ is the maximum feasible, or implementable, consumption level for each θ. Furthermore, the salvage value $\phi\left(x\left(\underline{\theta}\right), \underline{\theta}\right)$ is maximized by choosing $x\left(\underline{\theta}\right) = x^*\left(\underline{\theta}, \beta; p\right)$. Hence, for the rational consumer, the optimal solution for the rational consumer is indeed to implement $x\left(\theta\right) = x^*\left(\theta, \beta = 1; p\right) = x^o\left(\theta, p\right)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

On the other hand, this solution is never chosen by any (sophisticated) consumer with self control problems ($\beta < 1$). To show this, we use a proof by contradiction. Assume that a consumer with $\beta < 1$ implements a putative solution $x\left(\theta\right) = x^*\left(\theta, \beta; p\right)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Then consider the following deviation: in a (small) neighborhood of $\overline{\theta}$, consumption $x\left(\theta\right)$ is lowered by a small amount dx. Since $x^*\left(\theta, \beta; p\right) < x^*\left(\overline{\theta}, \beta; p\right)$ for all $\theta < \overline{\theta}$, the deviation can be implemented without violating the second-order incentive constraint (19), as long as dx is small enough (in abosolute value). Using (26) one can see that lowering consumption $x\left(\theta\right)$ by dx changes the consumer's objective function (J) by

$$\left[v'\left(x\left(\theta\right)\right) \frac{1}{\lambda\left(\theta\right)} - \left(1 - \beta\right) \delta h'\left(x\left(\theta\right)\right) \right] f\left(\theta\right) d\theta dx \tag{47}$$

But in the (small) neighborhood of $\overline{\theta}$, $1/\lambda\left(\theta\right)$ is very small (since $\lim_{\theta \to \overline{\theta}} \lambda\left(\theta\right) = \infty$) and hence the expression (47) is positive when $dx < 0$. Hence the deviation improves the ex ante consumer's (expected) welfare, and the putative solution ($x\left(\theta\right) = x^*\left(\theta, \beta; p\right)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$) cannot be optimal for a consumer with self control problems.

ii) *Maximum commitment.* The ex ante consumer's optimal solution is such that the constraint (28) binds for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. The consumer can implement this maximum-commitment solution with sin licenses, by choosing a small quota $y = x^*\left(\underline{\theta}, \beta; q\right)$, so that the consumer's ex post choice will be always based on the maximum (unit) price q. Essentially this then means that the ex ante consumer chooses to implement minimum feasible consumption $x^*\left(\theta, \beta; q\right)$ for all realizations $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Next we show that this solution may be chosen by a consumer with self control problems ($\beta < 1$), if $x^*\left(\underline{\theta}, \beta; q\right) = 0$, and $\Gamma \leq 0$, where Γ is given by (88); these conditions are the same as those that characterize the corner solution $y = 0$ under the system of sin licenses - see Appendix B2.

To prove these results, assume that a consumer implements a putative solution $x\left(\theta\right) = x^*\left(\theta, \beta; q\right)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Assume first that $x^*\left(\underline{\theta}, \beta; q\right) > 0$; in words, minimum feasible consumption is non-zero. Now consider the following deviation to the the putative solution: $x\left(\underline{\theta}\right)$ is increased by a small amount dx. Since (in this case where $x^*\left(\underline{\theta}, \beta; q\right) > 0$), $x^*\left(\theta, \beta; q\right) > x^*\left(\underline{\theta}, \beta; q\right)$ for $\theta > \underline{\theta}$, this deviation can be implemented without violating the second-order incentive constraint (19), if dx is small enough. Using (26) one can see that the effect of the deviation on the ex ante consumer's (expected) welfare (J) is

$$\frac{\partial \phi\left(x^*\left(\underline{\theta}, \beta; q\right), \underline{\theta}\right)}{\partial x} = \left[\underline{\theta} v'\left(x^*\left(\underline{\theta}, \beta; q\right)\right) - \beta \delta h'\left(x^*\left(\underline{\theta}, \beta; q\right)\right) - p\right] > 0$$

Hence, the deviation improves the ex ante consumer's (expected) welfare. We can then conclude that if $x^*\left(\underline{\theta}, \beta; q\right) > 0$, the putative solution ($x(\theta) = x^*(\theta, \beta; q)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$) cannot be optimal for the consumer.

Next, assume that $x^*(\theta, \beta; q) = 0$ for $\theta \in [\underline{\theta}, \theta_2]$, where $\theta_2 = \theta_2(0, \beta; q) = \frac{\beta\delta h'(0)+q}{v'(0)}$, while $x^*(\theta, \beta; q) > 0$ for $\theta > \theta_2$. Let us then consider the following deviation: we increase consumption by a small amount dx for all shock realizations $\theta \in [\underline{\theta}, \theta_2]$. This deviation can be implemented without violating the second-order incentive constraint (19). Using (26) one can see that the effect of the deviation on the ex ante consumer's (expected) welfare (J) is

$$\begin{aligned} \frac{dJ}{dx} &= \int_{\underline{\theta}}^{\theta_2} \left[v'(0) \frac{1}{\lambda(\theta)} - (1-\beta)\delta h'(0)\right] f(\theta)\, d\theta + \frac{\partial \phi(0,\underline{\theta})}{\partial x} \\ &= \int_{\underline{\theta}}^{\theta_2} \left[v'(0) \frac{1}{\lambda(\theta)} - (1-\beta)\delta h'(0)\right] f(\theta)\, d\theta + \underline{\theta} v'(0) - \beta\delta h'(0) - p \end{aligned} \tag{48}$$

If $\frac{dJ}{dx} > 0$, the deviation improves the ex ante consumer's (expected) welfare, and the putative strategy cannot be optimal for the consumer. However, if $\frac{dJ}{dx} \leq 0$ or

$$\int_{\underline{\theta}}^{\theta_2} \left[(1-\beta)\delta h'(0) - v'(0)\frac{1}{\lambda(\theta)}\right] f(\theta)\, d\theta \geq \underline{\theta} v'(0) - \beta\delta h'(0) - p$$

we cannot rule out the possibility that implementing $x(\theta) = x^*(\theta, \beta; q)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ is indeed optimal. Finally, using exactly the same steps as in Appedix B3, item a) one can show that (48) can be re-expressed as (88).

A2. Proof of Proposition 3

Benchmark case 1: The social planner chooses a pricing scheme for each consumer

The (utilitarian) social planner maximizes

$$W = E_i\left[E_{\theta_i}\left[V_i(\theta_i)\right]\right] = \int_i \int_{\underline{\theta}}^{\overline{\theta}} V_i(\theta_i) f_i(\theta_i)\, d\theta_i di \tag{49}$$

(where the inner expectation is over shock realizations θ_i of consumer i, and the outer expectation is over consumers i) subject to the incentive constraints (16) and (19), the pricing constraints (12) and (13), and the government budget constraint

$$E_i\left[\Pi_i\right] = E_i\left[E_{\theta_i}\left[T_i(\theta_i) - x_i(\theta_i)\right]\right]. \tag{50}$$

Plugging (10) and (50) into (49) shows that the government ends up maximizing

$$W = E_i\left[E_{\theta_i}\left[V_i\left(\theta_i\right)\right]\right] = E_i\left[E_{\theta_i}\left[\theta_i v_i\left(x_i\left(\theta_i\right)\right) - \delta_i h_i\left(x_i\left(\theta_i\right)\right) - x_i\left(\theta_i\right)\right]\right] + B \tag{51}$$

subject to (16) and (19), (12) and (13). Since (in this hypothetical thought experiment) the planner chooses a personalized pricing scheme / mechanism for each consumer, the planner's objective for an individual consumer i is

$$W_i = E_{\theta_i}\left[\theta_i v_i\left(x_i\left(\theta_i\right)\right) - \delta_i h_i\left(x_i\left(\theta_i\right)\right) - x_i\left(\theta_i\right)\right] + B. \tag{52}$$

Essentially, the planner does not care about the redistributive effects of the pricing schemes: one consumer's monetary loss is an other consumer's gain (through the system of subsidies). The planner just wants to implement (subject to constraints (16), (19), (12) and (13)) an allocation that is as close as possible to the (ex ante) first best for each consumer and for each preference shock realization.

Benchmark case 2: The consumer chooses the pricing scheme, but there are no redistributive effects Let us consider a hypothetical benchmark case, where pricing schemes $T_i(x)$ chosen by different consumers have no effects on the expected (re)distribution of income. Hence we assume

$$\Pi_i = \Pi_i(T_i) = E_{\theta_i}\left[T_i\left(x_i\left(\theta_i\right)\right) - x_i\left(\theta_i\right)\right] \tag{53}$$

In words, if the consumer chooses to pay a price exceeding the producer price, normalized to 1, he will get a subsidy from the government. Intuitively, the difference $T_i\left(x_i\left(\theta_i\right)\right) - x_i\left(\theta_i\right)$ can be thought of as a sin tax collected by the government. Equation (53) then shows that in expectation (or in the long run) the government returns all the tax revenues collected from consumer i to the consumer (however, notice that the subsidy does not depend on realized consumption $x_i\left(\theta_i\right)$). Then only the allocative - or corrective - effects of the scheme $T_i\left(x_i\left(\theta_i\right)\right)$ remain in this benchmark case.

Now, plugging (10) and (53) into (9) shows that, ex ante, the consumer maximizes

$$E_{\theta_i}\left[V_i\left(\theta\right)\right] = E_{\theta_i}\left[\theta_i v_i\left(x_i\left(\theta_i\right)\right) - \delta_i h_i\left(x_i\left(\theta_i\right)\right) - x_i\left(\theta_i\right)\right] + B = W_i \tag{54}$$

The consumer's objective function is therefore exactly the same as the planner's objective function (for consumer i); see expression (52).

Solving benchmark cases 1 and 2 We solve the optimization problem(s) in two steps: In the first we just treat (54), or alternatively and equivalently (52), as an unconstrained maximization problem, with $x_i\left(\theta_i\right)$ as a (sequence of) choice variable(s). The first-order conditions are

$$\theta_i v_i'\left(x_i^o\left(\theta_i\right)\right) - \delta_i h_i'\left(x_i^o\left(\theta_i\right)\right) - 1 = 0 \text{ for all } \theta_i. \tag{55}$$

In words, with each preference shock realization θ_i, one would like to choose, and implement, the first-best (ex ante) optimal consumption level $x_i^o(\theta_i)$.

In the second step, we check, whether, and to what extent, the first-best optimal solution can be implemented, given the constraints (16), (19), (12) and (13).

First, it is easy to see that the solution $x_i^o(\theta_i)$ has the property

$$\frac{dx_i^o(\theta_i)}{d\theta_i} = -\frac{v_i'(x_i^o(\theta_i))}{\theta_i v_i''(x_i^o(\theta_i)) - \delta_i h_i''(x_i(\theta_i))} \geq 0$$

Hence, the second-order incentive constraint (19) is satisfied.

Second, combining the first-order optimality constraint (55) and the first-order incentive constraint (16), we get

$$T_i^{o\prime}(x_i^o(\theta_i)) = \theta_i v_i'(x_i^o(\theta_i)) - \beta_i \delta_i h_i'(x_i^o(\theta_i)) = (1-\beta_i)\delta_i h_i'(x_i^o(\theta_i)) + 1 \qquad (56)$$

where T_i^o is the non-linear pricing scheme, that implements the first-best for the consumer. Now, the pricing scheme $T_i^{o\prime}(x_i^o(\theta_i))$ satisfies the constraint (12) if and only if $T_i^{o\prime}(x_i^o(\theta_i)) \geq p$. In particular, if $p = 1$ (the unit price floor p is set at the level of the producer price, equal to unity), the constraint (12) is never binding. On the other hand, if $p > 1$, (12) may be binding, for small values of the shock realization θ_i; then the best thing to do is to set $T_i'(x_i(\theta_i)) = p$ and realized consumption level is $x^*(\theta_i, \beta_i; p_i)$ is implicitly defined by (3).

Finally, the second pricing constraint (13) is not binding, for preference shock realization θ_i, if $T_i'(x_i^o(\theta_i)) \leq q$ while the constraint is binding, if $T_i'(x_i^o(\theta_i)) > q$. If constraint (13) is binding, the best one can do is to set $T_i'(x_i(\theta_i)) = q$ (this could be proved more formally by setting a Kuhn-Tucker optimization problem with inequality constraints).

To sum up, the pricing scheme is given by

$$T_i'(x_i(\theta_i)) = \max\left\{p,\ \min\left\{(1-\beta_i)\delta_i h_i'(x_i^o(\theta_i)) + 1, q\right\}\right\}. \qquad (57)$$

It is worth noting that the pricing scheme (57) can be presented with no explicit reference to shock realizations θ_i

$$T_i'(x_i) = \begin{cases} p & \text{for } x_i < x_{i1} \\ (1-\beta_i)\delta_i h'(x_i) + 1 & \text{for } x_i \in [x_{i1}, x_{i2}] \\ q & \text{for } x_i > x_{i2} \end{cases}$$

where x_{i1} and x_{i2} are implicitly defined by $(1-\beta_i)\delta_i h_i'(x_{i1}) = \tau_1$ and $(1-\beta_i)\delta_i h_i'(x_{i2}) = \tau_2$. Hence, implementing the socially optimal pricing scheme for a consumer i requires information on i) the consumer's self-control problems (the consumer's β_i), and the consumer's harm function ($\delta_i h_i'(x_i)$). The planner does *not* have know what kind of preference uncertainty the consumer faces. Quite clearly, however, the planner's solution (or equivalently the ex ante consumer's solution, with no redistributive effects), (57), cannot be implemented with a system of (generalized) sin licenses.

A3. Proof of Proposition 4

Let $\widehat{u}(x;\theta) = \theta v(x) - \delta h(x) - x$. Also notice that

$$\begin{aligned} \frac{\partial \widehat{u}(x^*(\theta,\beta;r),\theta)}{\partial x^*} &= \theta v'(x^*(\theta,\beta;r)) - \delta h'(x^*(\theta,\beta;r)) - 1 \\ &= -(1-\beta)\,\delta h'(x^*(\theta,\beta;r)) + (r-1), \quad r \in \{p,q,s\} \end{aligned} \tag{58}$$

where the second form is derived using the ex post consumer's first-order condition (3)

(i) Marginal sin tax on top of sin licenses. Assume that a system of sin licenses is in place. From the social point of view, consumer i's expected welfare is given by (see equation (52) above)

$$E_{\theta_i}[W_{\ell,i}(\theta_i;p,q)] = \int_{\underline{\theta}}^{\theta_1} \widehat{u}(x_i^*(\theta_i,\beta_i;p),\theta_i)\,dF_i(\theta_i)$$
$$+\int_{\theta_1}^{\theta_2} \widehat{u}(y_i,\theta_i)\,dF_i(\theta_i) + \int_{\theta_2}^{\overline{\theta}} \widehat{u}(x_i^*(\theta_i,\beta_i;q),\theta_i)\,dF_i(\theta_i)$$

Assume that initially $\tau_1 = 0$, so that $p = 1$ (with sin licenses the consumer can buy the sin good at the producer price), and that $0 < \tau_2 \leq k$ (and $q = 1 + \tau_2$). Let us now analyze what happens to consumer welfare, when p is raised by a small amount $d\tau_1$ (but q remains constant).

$$\begin{aligned} \frac{\partial E_{\theta_i}[W_{\ell,i}(\theta_i;p,q)]}{\partial p} &= -\int_{\underline{\theta}}^{\theta_1} \left[(1-\beta_i)\,\delta_i h_i'(x_i^*(\theta_i,\beta_i;p))\right] \frac{\partial x_i^*(\theta_i,\beta_i;p)}{\partial p} dF_i(\theta_i) \\ &\quad + [F_i(\theta_2) - F_i(\theta_1)]\left\{E[\theta_i \mid \theta_1 \leq \theta_i \leq \theta_2]\, v_i'(y_i) - \delta_i h_i'(y_i) - 1\right\} \frac{\partial y_i}{\partial p} \end{aligned} \tag{59}$$

(A change in p also affects the threshold values θ_1 and θ_2. However, the welfare effects operating through this channel cancel out each other.)

Next, using the consumer's first-order condition, related to the choice of the sin license quota y_i, (eq. (87)), we get

$$\begin{aligned} &[F_i(\theta_2) - F_i(\theta_1)]\left\{E[\theta_i \mid \theta_1 \leq \theta_i \leq \theta_2]\, v'(y_i) - \delta_i h_i'(y_i) - 1\right\} \\ =\ &-[1 - F_i(\theta_2)]\,\tau_2 \end{aligned}$$

Hence (59) can be re-expressed as

$$\begin{aligned}\frac{\partial E_{\theta_i}\left[W_{\ell,i}\left(\theta_i;p,q\right)\right]}{\partial p} &= -\int_{\underline{\theta}}^{\theta_1}\left[\left(1-\beta_i\right)\delta_i h_i'\left(x_i^*\left(\theta_i,\beta_i;p\right)\right)\right]\frac{\partial x_i^*\left(\theta_i,\beta_i;p\right)}{\partial p}dF_i\left(\theta_i\right) \\ &\quad -\left[1-F_i\left(\theta_2\right)\right]\tau_2\frac{dy_i}{dp}\end{aligned} \quad (60)$$

where $\frac{\partial x_i^*(\theta_i,\beta_i;p)}{\partial p} = \frac{1}{\theta_i v''\left(x_i^*\right)-\beta_i\delta_i h_i''(x^*)} < 0$. On the other hand, $\frac{dy_i}{dp} = \frac{\Psi}{V''(y_i)}$ where $\Psi \equiv v'\left(y_i\right)f\left(\theta_1\right)\left[\frac{1}{\lambda(\theta_1)}v'\left(y_i\right)-\left(1-\beta_i\right)\delta_i h_i'\left(y_i\right)\right]$. Now, we know that $V''\left(y_i\right)<0$, if there is an interior solution for y_i. Also, $\Psi \geq 0$ if there is an interior solution for y_i; this follows from the first-order condition (45), combined with the assumption $\lambda'\left(\theta_i\right)\geq 0$. Hence, $\frac{dy_i}{dp}\leq 0$, if there is an interior solution for y_i.

Finally, plugging these results into (60) yields $\frac{\partial E_{\theta_i}\left[W_{\ell,i}(\theta_i;p,q)\right]}{\partial p}\geq 0$ (with a strict inequality, if there is an interior solution in the sense that either $\theta_1>\underline{\theta}$ or $\theta_2<\overline{\theta}$). Hence, introducing a small sin tax on top of sin licenses improves social welfare (or at least does not lower welfare).

(ii) *Marginal sin licenses on top of a sin tax.* Assume that a uniform linear sin tax τ is in place, and consumption is given by $x^*\left(\theta_i,\beta_i;s\right)$, where $s=1+\tau$. From the social point of view, the consumer's expected welfare is given by

$$E_{\theta_i}\left[W_{\tau,i}\left(\theta,\beta;s\right)\right]=\int_{\underline{\theta}}^{\overline{\theta}}\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;s\right),\theta_i\right)dF_i\left(\theta_i\right)$$

Assume that a system of sin licenses is introduced: with a license the consumer can buy sin goods at unit price $p=s$, while without the license, the unit price of sin goods is $q=1+\tau_2>p$. From the social point of view, consumer i's expected welfare is given by

$$\begin{aligned}E_{\theta_i}\left[W_{\ell,i}\left(\theta,\beta;s,q\right)\right] &= \int_{\underline{\theta}}^{\theta_1}\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;s\right),\theta\right)dF_i\left(\theta_i\right) \\ &\quad +\int_{\theta_1}^{\theta_2}\widehat{u}\left(y_i,\theta_i\right)dF_i\left(\theta_i\right)+\int_{\theta_2}^{\overline{\theta}}\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;q\right),\theta_i\right)dF_i\left(\theta_i\right)\end{aligned}$$

Then from the social point of view, the introduction of sin licenses alters consumer i expected welfare by the amount

$$\begin{aligned}E_{\theta_i}\left[\Delta W_i\left(\theta,\beta;s,q\right)\right] &= E_{\theta_i}\left[W_{\ell,i}\left(\theta_i,\beta_i;s,q\right)\right]-E_{\theta_i}\left[W_{\tau,i}\left(\theta_i,\beta;s\right)\right] \\ &= \int_{\theta_1}^{\theta_2}\left[\widehat{u}\left(y_i,\theta_i\right)-\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;s\right),\theta_i\right)\right]dF_i\left(\theta_i\right) \\ &\quad +\int_{\theta_2}^{\overline{\theta}}\left[\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;q\right),\theta_i\right)-\widehat{u}\left(x_i^*\left(\theta_i,\beta_i;s\right),\theta_i\right)\right]dF_i\left(\theta_i\right)\end{aligned}$$

Next, let us study a system of *marginal* sin licenses, such that $\tau_2=\tau+d\tau$ and $q=s+d\tau$, where $d\tau$ is (very) small. Then θ_1, θ_2 and y are given by the equations (6), (94) and (95).

In particular, (94) implies that $d\theta_{21} \equiv \theta_2 - \theta_1 = \frac{d\tau}{v'(y_i)}$

Using these results, the social welfare effect of marginal sin licenses on consumer i can be expressed as

$$\begin{aligned}
&E_{\theta_i}\left[\Delta W_i\left(\theta_i, \beta_i; s, q\right)\right] = -\frac{\partial \widehat{u}\left(x_i^*\left(\theta_1, \beta_i; s\right), \theta_1\right)}{\partial x_i^*} \frac{\partial x_i^*\left(\theta_1, \beta; s\right)}{\partial \theta} dF\left(\theta_1\right) d\theta_{21} \\
&+\int_{\theta_2}^{\bar{\theta}} \frac{\partial \widehat{u}\left(x_i^*\left(\theta_i, \beta_i; s\right), \theta_i\right)}{\partial x_i^*} \frac{\partial x_i^*\left(\theta_i, \beta_i; s\right)}{\partial s} dF_i\left(\theta_i\right) d\tau \\
= \quad &\left[\left(1-\beta_i\right) \delta_i h_i'\left(x_i^*\left(\theta_i, \beta_i; q,\right)\right) - \tau\right] \frac{\partial x_i^*\left(\theta_1, \beta_i; s\right)}{\partial \theta} dF_i\left(\theta_1\right) \frac{d\tau}{v'\left(y\right)} \\
&-\int_{\theta_1}^{\bar{\theta}} \left[\left(1-\beta_i\right) \delta_i h_i'\left(x_i^*\left(\theta_i, \beta_i; s\right)\right) - \tau\right] \frac{\partial x_i^*\left(\theta_i, \beta_i; s\right)}{\partial s} dF_i\left(\theta_i\right) d\tau
\end{aligned}$$

However, the first effect is of an order of magnitude smaller than the second effect, and we can ignore it. Hence we can write

$$E_{\theta_i}\left[\Delta W_i\left(\theta_i, \beta_i; s, q\right)\right] = -\int_{\theta_1}^{\bar{\theta}} \left[\left(1-\beta_i\right) \delta_i h_i'\left(x_i^*\left(\theta_i, \beta_i; s\right)\right) - \tau\right] \frac{\partial x_i^*\left(\theta_i, \beta_i; s\right)}{\partial s} dF_i\left(\theta_i\right) d\tau \quad (61)$$

Finally, integrating over all consumers ($i \in I$) gives the aggregate welfare effect of marginal sin licenses. (Here we explicitly introduce the subindices i, to make clear the distinction between the individual level, and the aggregate level.)

$$\begin{aligned}
\Delta W &= E_i\left[E_\theta\left[\Delta W_i\left(\theta_i, \beta_i; s, q\right)\right]\right] \quad (62) \\
&= -\int_{i \in I} \int_{\theta_1\left(y_i; \beta_i, s\right)}^{\bar{\theta}} \left[\left(1-\beta_i\right) \delta_i h_i'\left(x^*\left(\theta_i, \beta_i; s\right)\right) - \tau\right] \frac{\partial x^*\left(\theta_i, \beta_i; s\right)}{\partial s} dF_i\left(\theta_i\right) dG\left(i\right) d\tau
\end{aligned}$$

($G\left(i\right)$ is the cumulative distributive function of consumers.)

Next, let us analyze the welfare effects of an alternative (marginal) policy reform: we increase the uniform linear sin tax τ by a small amount $d\tau$ (instead of introducing the system of marginal sin licenses). From the social point of view, the welfare effect of such a reform for consumer i is given by

$$\begin{aligned}
E_\theta\left[\Delta \widehat{W}_{\tau,i}\left(\theta_i, \beta_i; s\right)\right] &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\partial \widehat{u}\left(x_i^*\left(\theta_i, \beta_i; r\right), \theta_i\right)}{\partial x_i^*} \frac{\partial x_i^*\left(\theta_i, \beta_i; s\right)}{\partial s}\right] dF_i\left(\theta_i\right) d\tau \\
&= -\int_{\underline{\theta}}^{\bar{\theta}} \left[\left(1-\beta_i\right) \delta_i h_i'\left(x_i^*\left(\theta_i, \beta_i; s\right)\right) - \tau_1\right] \frac{\partial x^*\left(\theta_i, \beta_i; s\right)}{\partial s} dF_i\left(\theta_i\right) d\tau
\end{aligned}$$

Integrating over all consumers yields

$$\begin{aligned} \Delta\widehat{W}_{\tau} &= E_i\left[E_\theta\left[\Delta\widehat{W}_{\tau,i}\left(\theta_i,\beta_i;s\right)\right]\right] \\ &= -\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}\left[\left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i,\beta_i;s\right)\right)-\tau\right]\frac{\partial x^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i\left(\theta_i\right)dG\left(i\right)d\tau \end{aligned} \tag{63}$$

The final step in the proof is to compare the expressions (62) and (63), and to show that $\Delta\widehat{W}_{\tau}>0$ implies $\Delta W>0$. To do so, we rewrite (62) and (63) as follows:

$$\begin{aligned} \Delta W &= \left[\tau-\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}\left(1-\beta_i\right)\delta_i h_i'\left(x_i^*\left(\theta_i,\beta_i;p\right)\right)d\omega\left(\theta,i\right)\right]\times \\ &\quad \int_{i\in I}\int_{\theta_1\left(y_i;\beta_i,s\right)}^{\overline{\theta}}\frac{\partial x_i^*\left(\theta_i,\beta_i;p\right)}{\partial s}dF_i\left(\theta_i\right)dG\left(i\right)d\tau \end{aligned} \tag{64}$$

and

$$\begin{aligned} \Delta\widehat{W}_{\tau} &= \left[\tau-\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}\left(1-\beta_i\right)\delta_i h_i'\left(x_i^*\left(\theta_i,\beta_i;s\right)\right)d\widehat{\omega}\left(\theta,i\right)\right]\times \\ &\quad \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial x^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i\left(\theta_i\right)dG\left(i\right)d\tau \end{aligned} \tag{65}$$

The weighting function, or density functions, $d\omega\left(\theta,i\right)$ and $d\widehat{\omega}\left(\theta,i\right)$ are defined as follows

$$d\omega\left(\theta,i\right)=\begin{cases}\left(\dfrac{\frac{\partial x^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i(\theta_i)dG(i)}{\int_{i\in I}\int_{\theta_{1,i}\left(y_i;\beta_i,s\right)}^{\overline{\theta}}\frac{\partial x_i^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i(\theta_i)dG(i)d\tau}\right) & \text{if } \theta_i\geq\theta_{1,i}\left(y_i;\beta_i,s\right) \\ 0 & \text{if } \theta_i<\theta_{1,i}\left(y_i;\beta_i,s\right)\end{cases},$$

(where $\theta_{1,i}$ and y_i are characterized by (6) and (95)), and

$$d\widehat{\omega}\left(\theta,i\right)=\left(\frac{\frac{\partial x_i^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i\left(\theta_i\right)dG\left(i\right)}{\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial x^*\left(\theta_i,\beta_i;s\right)}{\partial s}dF_i\left(\theta_i\right)dG\left(i\right)d\tau}\right)$$

Clearly, $\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}d\omega\left(\theta,i\right)=\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}}d\widehat{\omega}\left(\theta,i\right)=1$.

Now, we can prove that $\Delta\widehat{W}_{\tau}>0$ implies $\Delta W>0$. This is done in four steps. First,it is easy to see that the distribution $\omega\left(\theta,i\right)$ stochastically dominates the distribution $\widehat{\omega}\left(\theta,i\right)$: basically $\omega\left(\theta,i\right)$ is obtained from $\widehat{\omega}\left(\theta,i\right)$ by left-truncation, i.e. small values of θ (that is values $\theta<\theta_{1,i}\left(y_i;\beta_i,p\right)$) have been left out of the distribution $\omega\left(\theta,i\right)$; moreover the truncation point $\theta_{1,i}\left(y_i;\beta_i,p\right)$ is higher, ceteris paribus, for relatively rational consumers (high β_i) than for consumers with severe self-control problems (low β_i). Second, the self-control wedge $\left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i,\beta_i;p\right)\right)$ is clearly increasing (or non-decreasing) in the preference shock θ_i, and in the severity of self-control problems $\rho_i\equiv\left(1-\beta_i\right)$. Third, combining steps one and two, one can conclude that

$$\left[\tau - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} (1-\beta_i)\,\delta_i h_i'\left(x_i^*\left(\theta_i,\beta_i;s\right)\right) d\omega\left(\theta,i\right)\right] < \left[\tau - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} (1-\beta_i)\,\delta_i h_i'\left(x_i^*\left(\theta_i,\beta_i;s\right)\right) d\widehat{\omega}\left(\theta,i\right)\right] \tag{66}$$

Fourth, since $\int_{i\in I}\int_{\theta_1(y_i;\beta_i,s)}^{\overline{\theta}} \frac{\partial x_i^*(\theta_i,\beta_i;s)}{\partial s} dF_i\left(\theta_i\right) dG\left(i\right) d\tau < 0$ and $\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial x_i^*(\theta_i,\beta_i;s)}{\partial s} dF_i\left(\theta_i\right) dG\left(i\right) d\tau < 0$, we can conclude that $\Delta\widehat{W}_\tau > 0$ implies $\Delta W > 0$. This conclusion follows from expressions (64) and (65), together with the inequality (66).

A4. Proof of Proposition 5

Let $\widehat{u}_i\left(x_i;\theta_i\right) = \theta_i v_i\left(x_i\right) - \delta_i h_i\left(x_i\right) - x_i$. A sophisticated consumer i chooses a (generalized) system of sin licenses, with price floor $p = 1+\tau_1$ and price ceiling $q = 1+\tau_2$ rather than a linear sin tax τ and a linear consumer price $s = 1+\tau$ if and only if this improves the individual's expected welfare measure

$$\Delta V_i \equiv E_{\theta_i}\left[\Delta V_i\left(\theta_i,\beta_i;p,q,s\right)\right] \equiv E_{\theta_i}\left[V_{\ell,i}\left(\theta_i,\beta_i;p,q\right)\right] - E_{\theta_i}\left[V_{\tau,i}\left(\theta_i,\beta_i;s\right)\right] > 0 \tag{67}$$

where

$$V_{\ell,i}\left(\theta_i,\beta_i;p,q\right) = \widehat{u}_i\left(x_{\ell,i};\theta_i\right) - \tau_1 x_{\ell,i} - \tau_2 x_i^q + B + \Pi \tag{68}$$

is the consumer's welfare measure under (generalized) sin licenses and

$$V_{\tau,i}\left(\theta_i,\beta_i;s\right) = \widehat{u}_i\left(x_{\tau,i};\theta_i\right) - \tau x_{\tau,i} + B + \Pi \tag{69}$$

is the consumer's welfare measure under the linear sin tax. Here $x_{\ell,i} = x_{\ell,i}\left(\theta_i,\beta_i;p,q\right)$ refers to individual i's consumption under the system of licenses, $x_i^q = x_i^q\left(\theta_i,\beta_i;p,q\right) = \max\left\{0, x_{\ell,i}\left(\theta_i,\beta_i;p,q\right) - y_i\right\}$), while $x_{\tau,i} = x_{\tau,i}\left(\theta_i,\beta_i;s\right)$ refers to his consumption under sin taxes. Also notice that the lump-sum transfer Π the consumer receives from the government is the same under both schemes: The consumer is choosing the pricing scheme for himself, not for the whole economy; the lump-sum transfer does not depend on the choice made by the individual consumer.

Meanwhile, if the consumer shifts from the linear tax to sin licenses, this implies a change in the social (planner's) welfare measure equal to

$$\Delta W_i \equiv E_{\theta_i}\left[\Delta W_i\left(\theta_i,\beta_i;p,q,s\right)\right] = E_{\theta_i}\left[W_{\ell,i}\left(\theta_i,\beta_i;p,q\right) - W_{\tau,i}\left(\theta_i,\beta_i;p,q\right)\right] \tag{70}$$

where

$$W_{\ell,i}\left(\theta_i,\beta_i;p,q\right) = \widehat{u}_i\left(x_{\ell,i};\theta_i\right) + B \tag{71}$$

is the social (planner's) welfare measure under (generalized) sin licenses, while

$$W_{\tau,i}(\theta_i, \beta_i; s) = \widehat{u}_i(x_{\tau,i}; \theta_i) + B \tag{72}$$

is the social (planner's) welfare measure under the linear sin tax. Remember that the planner does not care about the redistributive effects of the regulatory schemes. See the discussion in Section 4.2; see also Appendix A2.

Next, by inspecting the equations (68) - (72), one can conclude that

$$\Delta W_i = \Delta V_i + \Delta TR_i \tag{73}$$

where

$$\Delta TR_i = E_{\theta_i}[\Delta TR_i(\theta_i, \beta_i; p, q, s)] = E_{\theta_i}[TR_{\ell,i}(\theta_i, \beta_i; p, q) - TR_{\tau,i}(\theta_i, \beta_i; s)]$$

is the change in the consumer's tax bill, which is equal to difference of the consumer's tax payments under the system of sin licenses, $TR_{\ell,i}(\theta_i, \beta_i; p, q) = \tau_1 x_{\ell,i}(\theta_i, \beta_i; p, q) + \tau_2 x_i^q(\theta_i, \beta_i; p, q)$ and his tax payments under the linear sin tax $TR_{\tau,i}(\theta_i, \beta_i; s) = \tau x_{\tau,i}(\theta_i, \beta_i; s)$.

Evaluating the aggregate consequences of the reform involves integrating (or summing) over all consumers, who choose to switch from taxes to licenses, let us call this group of consumers group L. Now from equation (73) we get

$$\Delta W \equiv E_{i \in L}[\Delta W_i] = E_{i \in L}[\Delta V_i] + E_{i \in L}[\Delta TR_i] \tag{74}$$

Next, (67) implies that $E_{i \in L}[\Delta V_i] > 0$. Hence we know that social welfare is guaranteed to increase ($\Delta W > 0$) if (sufficient condition)

$$\Delta TR = E_{i \in L}[\Delta TR_i] \geq 0$$

where ΔTR is the change in the government's aggregate tax income, following the reform.

Appendix B: Additional results and derivations

B1. Appendix to Section 3.1

Let us consider the demand for sin licenses under preference uncertainty, but no potential secondary market trade. The consumer chooses the amount of sin licenses y so as to maximize

$$\begin{aligned} V(y) &= \int_{\underline{\theta}}^{\theta_1} \left[\theta v\left(x^*(\theta,\beta;p)\right) - \delta h\left(x^*(\theta,\beta;p)\right) - p x^*(\theta,\beta;p)\right] dF(\theta) \\ &\quad + \int_{\theta_1}^{\overline{\theta}} \left[\theta v(y) - \delta h(y) - py\right] dF(\theta) \end{aligned}$$

where $\theta_1 = \theta_1(y,\beta;p)$ is given by (6) and $x^*(\theta,\beta;p)$ is given by (3).

Let us begin by considering fully rational consumers. Since $x^*(\theta,\beta=1;p) = x^o(\theta;p)$ is the consumption level that maximizes ex ante utility for each shock realization θ, it is immediately clear that a fully rational consumer chooses a quota $y \geq x^*\left(\overline{\theta},\beta=1;p\right) = x^o\left(\overline{\theta};p\right)$ that never binds.

Our main analysis concerns consumers with self-control problems $\beta < 1$. The first derivative of the ex ante value function $V(y)$ is

$$V'(y) = \left[1 - F\left(\theta_1(y,\beta;p)\right)\right]\left(E\left[\theta \mid \theta \geq \theta_1(y,\beta;p)\right] v'(y) - \delta h'(y) - p\right) \tag{75}$$

Notice that $x^*(\theta_1,\beta;p) = y$; hence the terms involving $\frac{\partial \theta_1(y,\beta;p)}{\partial y}$ cancel out.

Interior solution $y \in (0, x^{\max})$

The minimum consumption of sin goods is 0, while the maximum consumption is $x^{\max} = x^*\left(\overline{\theta},\beta;p\right)$. From (6) it is clear that $\theta_1 \to \overline{\theta}$ if $y \to x^{\max}$. Since $(1 - F(\theta_1)) \geq 0$, (75) implies that the sign of $\lim_{y \to x^{\max}} V'(y)$ is the same as the sign of

$$\overline{\theta} v'\left(x^*\left(\overline{\theta},\beta;p\right)\right) - \delta h'\left(x^*\left(\overline{\theta},\beta;p\right)\right) - p = -(1-\beta)\,\delta h'\left(x^*\left(\overline{\theta},\beta;p\right)\right) \tag{76}$$

Hence, $\lim_{y \to x^{\max}} V'(y) < 0$ if $\beta < 1$ (while $\lim_{y \to x^{\max}} V'(y) = 0$ if $\beta = 1$). On the other hand, if $y = 0$, it is clear that $\theta_1(0,\beta;p) \leq \underline{\theta}$. Then using (75) we get

$$V'(0) = E[\theta]\, v'(0) - \delta h'(0) - p$$

Now, since $V'(y)$ is continuous over $y \in [0, x^{\max})$, we can conclude that if $V'(0) > 0$, there exists an interior maximum $y \in (0, x^{\max})$, which is characterized by the first-order condition $V'(y) = 0$ and the second-order condition $V''(y) < 0$. Using (75), the first-order condition can be further re-expressed as

$$E\left[\theta \mid \theta \geq \theta_1\right] v'(y) - \delta h'(y) - p = 0. \tag{77}$$

Notice that if the optimal solution is such that $y < x^* (\underline{\theta}, \beta; p)$, we have $\theta_1 < \underline{\theta}$ and the first-order condition boils down to

$$E[\theta] v'(y) - \delta h'(y) - p = 0. \tag{78}$$

Finally, for the condition to hold $V'(0) > 0$ to hold, and for an interior solution $y \in (0, x^{\max})$ to exist, we must have $E[\theta] v'(0) - \delta h'(0) - p > 0$. This condition can be also (re)expressed as $x^o (E[\theta], p) > 0$. In words, an interior solution $y \in (0, x^{\max})$ is guaranteed to exist, if the rational consumption level for the average preference shock realization, $E[\theta]$, is non-zero.

Corner solution $y = 0$

Hence we know that if $V'(0) > 0$, there exists an interior maximum $y \in (0, x^{\max})$, and the consumer chooses a non-zero sin license quota. What about the case, where $V'(0) \leq 0$? Is it optimal for the consumer to ask for a zero quota, $y = 0$, meaning that consumer fully abstains from consuming the sin good?

This is indeed the case, if we can show that the function $V(y)$ can cross the x-axis at most once. To do so, we next study more carefully the second-order condition: if $V''(y^*) < 0$ for any y^* such that $V'(y^*) = 0$, we can conclude that there is at most one solution to the equation $V'(y^*) = 0$. Then $V'(0) < 0$ and $\lim_{y \to x^{\max}} V'(y) < 0$ imply that there are no solutions to the equation $V'(y^*) = 0$, and $V'(y) < 0$ for all $y \in [x^{\min}, x^{\max})$. On the other hand $V'(0) = 0$ implies that $y = 0$ is the only solution. In both cases, $y = 0$ is the optimal choice for the consumer.

The second-order condition can be expressed as

$$V''(y) = E[\theta] v''(y) - \delta h''(y) - p < 0 \tag{79}$$

if $y < x^* (\underline{\theta}, \beta; p)$. On the other hand,

$$V''(y) = \int_{\theta_1}^{\overline{\theta}} \left[\theta v''(y) - \delta h''(y)\right] f(\theta) \, d\theta - \left[\theta_1 v'(y) - \delta h'(y) - p\right] f(\theta_1) \frac{\partial \theta_1}{\partial y}. \tag{80}$$

if $y \geq x^* (\underline{\theta}, \beta; p)$. Using (6), together with

$$\frac{\partial \theta_1 (y, \beta; p)}{\partial y} = \frac{\beta \delta h''(y) - \theta_1 v''(y)}{v'(y)} > 0 \tag{81}$$

(implied by (6)) and $(1 - \beta) \frac{\delta h'(y)}{v'(y)} = \left(\int_{\theta_1}^{\overline{\theta}} (1 - F(\theta)) \, d\theta\right) (1 - F(\theta_1))^{-1}$ (implied by the (77)) allows us to rewrite the second-order condition for $y \geq x^* (\underline{\theta}, \beta; p)$ (80) as

$$\Upsilon \equiv \int_{\theta_1}^{\overline{\theta}} \left[\left\{v''(y) \left[\theta \lambda(\theta) - \theta_1 \lambda(\theta_1)\right] - \delta h''(y) \left[\lambda(\theta) - \beta \lambda(\theta_1)\right]\right\} (1 - F(\theta))\right] d\theta, \tag{82}$$

where $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is the hazard rate. Now, from (82) it is clear that the second-order condition holds for $y \geq x^*(\underline{\theta}, \beta; p)$ (i.e. $\Upsilon \leq 0$) if $\lambda'(\theta) \geq 0$ for all $\theta \in [\theta_1, \overline{\theta}]$ (sufficient condition).

In sum, if $\lambda'(\theta) \geq 0$, there is at most one solution ($y = y^*$) to the equation $V'(y) = 0$. If this condition holds, $V'(0) \leq 0$ implies that a zero quota ($y = 0$) is the optimal choice for the consumer. (Also recall that if $V'(0) > 0$, there exists an interior maximum $y \in (0, x^{\max})$, and the consumer chooses a non-zero sin license quota.)

Comparative statics: the size of the quota and the degree of self-control problems

Next we study how the size of the quota y depends on the degree of self-control problems β. Assume that an interior solution $y \in (0, x^{\max})$ exists. Some straightforward algebra shows that

$$\frac{\partial y}{\partial \beta} = -\frac{2f(\theta_1)(1-\beta)(\delta h'(y))^2}{v'(y)V''(y)} \tag{83}$$

Clearly, $\frac{\partial y}{\partial \beta} > 0$ if $\theta_1 \geq \underline{\theta}$ and $y \geq x(^*\underline{\theta}, \beta; p)$ (here we assume that $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta})$); hence more rational consumers (with a higher β) choose a larger quota . However notice that if $\theta_1 < \underline{\theta}$ and $y < x(^*\underline{\theta}, \beta; p)$ we have $\frac{\partial y}{\partial \beta} = 0$ (since $f(\theta_1) = 0$ when $\theta_1 < \underline{\theta}$) and all consumers (with severe self-control problems) for whom $y < x^*(\underline{\theta}, \beta; p)$, choose the same y, irrespectively of the exact value of β; a non-zero quota $0 < y < x^*(\underline{\theta}, \beta; p)$ is characterized by the first-order condition (78).

The size of the quota and the rational level of consumption

To further characterize the size of the sin license quota y, we next compare y to the rational level of consumption. Denote the rational level of consumption, with preferences θ and consumer price p, by $x^o(\theta; p)$. This level of consumption is given by

$$\theta v'(x^o(\theta; p)) - \delta h'(x^o(\theta; p)) - p = 0 \tag{84}$$

In particular, we show that a consumer with self-control problems ($\beta < 1$) chooses a quota $y < x^o(\overline{\theta}; p)$, where $x^o(\overline{\theta}; p)$ is the highest possible rational level of consumption at price p (corresponding to the highest possible preference shock realization $\overline{\theta}$).

To show that $y^*(\beta; p) < x^o(\overline{\theta}; p)$, we assume by contrast that $y^*(\beta; p) = x^o(\overline{\theta}; p)$. From equation (84) we get $\delta h'(x^o(\overline{\theta}; p)) + p = \overline{\theta} v'(x^o(\overline{\theta}; p))$. Using this result, the left-hand side of the first-order condition (77) takes the form

$$\left(E\left[\theta \mid \theta \geq \theta_1\left(x^o(\overline{\theta}; p), \beta; p\right)\right] - \overline{\theta}\right) v\left(x^o(\overline{\theta}; p)\right).$$

On the other hand, by (6)

$$\theta_1\left(x^o(\overline{\theta}; p); \beta\right) = \frac{\beta\delta h'\left(x^o(\overline{\theta}; p)\right) + p}{v'\left(x^o(\overline{\theta}; p)\right)} \tag{85}$$

Then clearly $\theta_1\left(x^o\left(\overline{\theta};p\right),\beta;p\right) < \overline{\theta}$: if $\theta_1\left(x^o\left(\overline{\theta}\right);\beta\right)$ were equal to $\overline{\theta}$, (85) would imply $\overline{\theta}v'\left(x^o\left(\overline{\theta};p\right)\right) - \beta\delta h'\left(x^o\left(\overline{\theta};p\right)\right) + p = 0$; but this contradicts the first-order condition characterizing the optimal choice $x^o\left(\overline{\theta};p\right)$ (look at equation (84) with $\theta = \overline{\theta}$). Finally, since $\theta_1\left(x^o\left(\overline{\theta}\right),\beta;p\right) < \overline{\theta}$, yields $\left(E\left[\theta \mid \theta \geq \theta_1\left(x^o\left(\overline{\theta}\right),\beta;p\right)\right] - \overline{\theta}\right) v\left(x^o\left(\overline{\theta}\right)\right) < 0$: assuming $y^*\left(\beta;p\right) = x^o\left(\overline{\theta};p\right)$ leads to a contradiction. Thus the optimal choice of $y^*\left(\beta;p\right)$ must be lower than $x^o\left(\overline{\theta};p\right)$.

B2. Appendix to Section 3.2

Assume that without sin licenses, the consumer can buy sin goods at unit price q. The consumer then chooses the amount of sin licenses y so as to maximize

$$\begin{aligned} V\left(y\right) &= \int_{\underline{\theta}}^{\theta_1} \left[\theta v\left(x^*\left(\theta,\beta;p\right)\right) - \delta h\left(x^*\left(\theta,\beta;p\right)\right) - p x^*\left(\theta,\beta;p\right)\right] dF\left(\theta\right) \\ &+ \int_{\theta_1}^{\theta_2} \left[\theta v\left(y\right) - \delta h\left(y\right) - p y\right] dF\left(\theta\right) \\ &\int_{\theta_2}^{\overline{\theta}} \left[\theta v\left(x^*\left(\theta,\beta;q\right)\right) - \delta h\left(x^*\left(\theta,\beta;q\right)\right) - p y - q(x^*\left(\theta,\beta;q\right) - y)\right] dF\left(\theta\right). \end{aligned}$$

subject to (6) and (7). The derivative of the value function with respect to y is

$$\begin{aligned} V'\left(y\right) = \left[F\left(\theta_2\right) - F\left(\theta_1\right)\right]\left[E\left[\theta \mid \theta_2 \geq \theta \geq \theta_1\right] v'\left(y\right) - \delta h'\left(y\right) - p\right] \\ + \left[1 - F\left(\theta_2\right)\right]\left(q - p\right). \end{aligned} \quad (86)$$

This expression is obtained noting that $x^*\left(\theta_1^*,\beta;p\right) = y$ and $x^*\left(\theta_2,\beta;q\right) = y$; hence the terms involving $\frac{\partial\theta_1(y,\beta;p)}{\partial y}$ and $\frac{\partial\theta_2(y;\beta;q)}{\partial y}$ cancel out.

Interior solution $y \in (0, x^{\max})$

We show that an interior solution $y \in (0, x^{\max})$ exists if either i) $x^*\left(\underline{\theta},\beta;q\right) > 0$ or ii) $x^o\left(\underline{\theta},p\right) > 0$ (sufficient conditions). These conditions are rather intuitive: Item i) essentially means that the consumer cannot commit to zero consumption, for any preference shock realization $\theta \in \left[\underline{\theta},\overline{\theta}\right]$ (due to potential secondary market trade). Item ii) means that it cannot be in the consumer's interests to commit to zero consumption, even if this option is available, since the rational consumption level $x^o\left(\theta,p\right)$ is strictly positive for all shock realizations $\theta \in \left[\underline{\theta},\overline{\theta}\right]$.

The minimum consumption of sin goods is $x^{\min} = x^*\left(\underline{\theta},\beta;q\right)$, while the maximum consumption is $x^{\max} = x^*\left(\overline{\theta},\beta;p\right)$. From (6) and (7) it clearly follows that $\theta_1 \to \overline{\theta}$ and $\theta_2 > \overline{\theta}$ if $y \to x^{\max}$. Since $F\left(\theta_2\right) - F\left(\theta_1\right) \geq 0$, the sign of $\lim_{y\to x^{\max}} V'\left(y\right)$ is the same as the sign of (76). Hence, $\lim_{y\to x^{\max}} V'\left(y\right) < 0$ if $\beta < 1$ (while $\lim_{y\to x^{\max}} V'\left(y\right) = 0$ if $\beta = 1$).

At the lower boundary $\left(y = x^{\min}\right)$ we need to tackle two subcases.

i) First, if $y = x^{\min} = x^*(\underline{\theta}, \beta; q) > 0$, it follows from (6) and (7) that $\theta_2\left(x^{\min}, \beta; q\right) = \underline{\theta}$ while $\theta_1\left(x^{\min}, \beta; p\right) < \underline{\theta}$. Plugging these results into (86) yields

$$V'\left(x^{\min}\right) = q - p > 0.$$

Now, since $V'\left(x^{\min}\right) > 0$, $\lim_{y \to x^{\max}} V'(y) < 0$, and $V'(y)$ is continuous over $y \in [x^{\min}, x^{\max})$, there exists an interior maximum $y \in (x^{\min}, x^{\max})$, which is characterized by the first-order condition $V'(y) = 0$ and the second-order condition $V''(y) < 0$. Using (86), the first-order condition can be re-expressed as

$$\begin{aligned} V'(y) = [F(\theta_2) - F(\theta_1)]\left[E[\theta \mid \theta_2 \geq \theta \geq \theta_1] v'(y) - \delta h'(y) - p\right] \\ + [1 - F(\theta_2)](q - p) = 0 \end{aligned} \tag{87}$$

ii) Second, if $y = x^{\min} = x^*(\underline{\theta}, \beta; q) = 0$, we have $\theta_2(0, \beta; q) > \underline{\theta}$; this follows from (7), since in this case $\underline{\theta} v'(0) - \beta\delta h'(0) - q < 0$.On the other hand since $y = 0$, we clearly have $\theta_1(0, \beta; p) \leq \underline{\theta}$; no matter what the shock realization θ, the consumer cannot buy an amount of the sin good that would fall short of the zero quota. Then using (86) we get

$$\begin{aligned} V'\left(x^{\min}\right) &= V'(0) \\ &= F(\theta_2(0, \beta; q))\left(E[\theta \mid \theta \leq \theta_2(0, \beta; q)] v'(0) - \delta h'(0) - p\right) \\ &+ [1 - F(\theta_2(0, \beta; q))](q - p) \end{aligned} \tag{88}$$

Now, since $\lim_{y \to x^{\max}} V'(y) < 0$ and since $V'(y)$ is continuous over $y \in [x^{\min}, x^{\max})$, we can conclude that if $V'(0) > 0$, there exists an interior maximum $y \in (0, x^{\max})$, which is characterized by the first-order condition (87), together with the second-order condition $V''(y) < 0$. Finally, using (88) one can show that a *sufficient* condition for $V'(0) > 0$ to hold is that $\underline{\theta} v'(0) - \delta h'(0) - p > 0$, i.e. $x^o(\underline{\theta}, p) > 0$. In words, an interior solution $y \in [x^{\min}, x^{\max})$ is guaranteed to exist, if the rational consumption level $x^o(\theta, p)$ is non-zero for all shock realizations. However $V'(0) > 0$ may hold even when $\underline{\theta} v'(0) - \delta h'(0) - p \leq 0$ (i.e. $x^o(\underline{\theta}, p) = 0$).

Corner solution $y = 0$

Hence we know that if either $x^{\min} = x^*(\underline{\theta}, \beta; q) > 0$ or $V'(0) > 0$, there exists an interior maximum $y \in (0, x^{\max})$, and the consumer chooses a non-zero sin license quota. What about the case, where $x^{\min} = x^*(\underline{\theta}, \beta; q) = 0$ and $V'(0) \leq 0$? Do we then have a corner solution $y = 0$, meaning that the optimal choice for the consumer is to ask for a zero quota?

This is indeed the case, if we can show that the function $V'(y)$ can cross the x-axis at most once. Just like above in Appendix B1, we next study more carefully the second-order condition: if $V''(y^*) < 0$ for any y^* such that $V'(y^*) = 0$, we can conclude that there is at most one solution to the equation $V'(y^*) = 0$. Then $V'(0) < 0$ and $\lim_{y \to x^{\max}} V'(y) < 0$ imply that there are no solutions to the equation $V'(y^*) = 0$, while $V'(0) = 0$ implies that

$y = 0$ is the only solution. In both cases, $y = 0$ is the optimal choice for the consumer.

The second-order condition is of the form

$$V''(y) = \int_{\theta_1}^{\theta_2} \left[\theta f(\theta) v''(y) - f(\theta)\delta h''(y)\right] d\theta + \left[\theta_2 v'(y) - \delta h'(y) - q\right] f(\theta_2) \frac{\partial \theta_2}{\partial y}$$
$$- \left[\theta_1 v'(y) - \delta h'(y) - p\right] f(\theta_1) \frac{\partial \theta_1}{\partial y}. \tag{89}$$

where $\frac{d\theta_1}{\partial y}$ is given by (81) and

$$\frac{\partial \theta_2(y, \beta; q)}{\partial y} = \frac{\beta \delta h''(y)}{v'(y)} - \theta_2 \frac{v''(y)}{v'(y)} > 0$$

There are several subcases here.

a) Assume that $x^*\left(\overline{\theta}, \beta; q\right) < x^*\left(\underline{\theta}, \beta; p\right)$.

i) If $y \in \left[x^*\left(\underline{\theta}, \beta; q\right), x^*\left(\overline{\theta}, \beta; q\right)\right]$, we have $\theta_1 < \underline{\theta}$ and $\theta_2 \in \left[\underline{\theta}, \overline{\theta}\right]$. Then, since $f(\theta_1) = 0$ we get

$$V''(y) = \int_{\theta_1}^{\theta_2} \left[\theta f(\theta) v''(y) - f(\theta)\delta h''(y)\right] d\theta - (1-\beta)\delta h'(y) f(\theta_2) \frac{\partial \theta_2}{\partial y} < 0 \tag{90}$$

ii) $y \in \left[x^*\left(\overline{\theta}, \beta; q\right), x^*\left(\underline{\theta}, \beta; p\right)\right]$, we have $\theta_1 < \underline{\theta}$ and $\theta_2 > \overline{\theta}$. Then since $f(\theta_1) = f(\theta_2) = 0$ we get

$$V''(y) = \int_{\theta_1}^{\theta_2} \left[\theta f(\theta) v''(y) - f(\theta)\delta h''(y)\right] d\theta < 0$$

iii) If $y \in \left[x^*\left(\underline{\theta}, \beta; p\right), x^*\left(\overline{\theta}, \beta; p\right)\right]$, we have $\theta_1 = \left[\underline{\theta}, \overline{\theta}\right]$ and $\theta_2 > \overline{\theta}$. Then since $f(\theta_2) = 0$, the second-order condition (89) takes the form (80); as shown in Appendix B.1, the second-order condition (80) holds if $\lambda'(\theta) \geq 0$ for all $\theta \in \left[\theta_1, \overline{\theta}\right]$ (sufficient condition).

b) Assume that $x^*\left(\overline{\theta}, \beta; q\right) > x^*\left(\underline{\theta}, \beta; p\right)$.

i) If $y \in [x^*\left(\underline{\theta}, \beta; q\right), x^*\left(\underline{\theta}, \beta; p\right))$, we have $\theta_1 < \underline{\theta}$ and $\theta_2 \in \left[\underline{\theta}, \overline{\theta}\right]$. Then, since $f(\theta_1) = 0$, $V''(y)$ takes the form (90), and the second-order condition holds.

ii) If $y \in \left[x^*\left(\underline{\theta}, \beta; p\right), x^*\left(\overline{\theta}, \beta; q\right)\right]$, we have $\theta_1 \in \left[\underline{\theta}, \overline{\theta}\right]$ and $\theta_2 \in \left[\underline{\theta}, \overline{\theta}\right]$. Next, using equations (6), (7) and (81) together with (implied by (7)) and $\frac{(1-\beta)\delta h'(y)}{v'(y)} = \int_{\theta_1}^{\theta_2} (1 - F(\theta))\, d\theta \left[F(\theta_2) - F(\theta_1)\right]^{-1}$ (implied by the first-order condition (87)), allows us to rewrite the second-order condition (89) as follows

$$V''(y) = \widehat{\Upsilon} = v''(y) \int_{\theta_1}^{\theta_2} \left\{\left[\theta \lambda(\theta) + \theta_2 \lambda(\theta_2) \Phi(\theta_2) - \theta_1 \lambda(\theta_1) \Phi(\theta_1)\right](1 - F(\theta))\right\} d\theta$$
$$-\delta h''(y) \int_{\theta_1}^{\theta_2} \left\{\left[\lambda(\theta) + \beta \lambda(\theta_2) \Phi(\theta_2) - \beta \lambda(\theta_1) \Phi(\theta_1)\right](1 - F(\theta))\right\} d\theta \tag{91}$$

where

$$\Phi(\theta_j) = \frac{1 - F(\theta_j)}{F(\theta_2) - F(\theta_1)}, \quad j = 1, 2$$

From (91) one can see that the second-order condition hinges on the hazard rate $\lambda(\theta)$. However, unlike in the case with no (potential) secondary market trade (see (82)), the second-order condition is not necessarily satisfied when $\lambda'(\theta) \geq 0$: the terms $\theta_1 \lambda(\theta_1) \Phi(\theta_1)$ and $\beta \lambda(\theta_1) \Phi(\theta_1)$ can potentially be so large (in absolute value) that they make expression (91) positive.

iii) If $y \in \left[x^*\left(\overline{\theta}, \beta; q\right), x^*\left(\overline{\theta}, \beta; p\right)\right]$, we have $\theta_1 = \left[\underline{\theta}, \overline{\theta}\right]$ and $\theta_2 > \overline{\theta}$. Then since $f(\theta_2) = 0$, the second-order condition (89) takes the form (80); as shown in Appendix B.1, the second-order condition (80) holds if $\lambda'(\theta) \geq 0$ for all $\theta \in \left[\theta_1, \overline{\theta}\right]$ (sufficient condition).

In sum, the function $V'(y)$ can cross the x-axis at most once, if a) $x^*\left(\overline{\theta}, \beta; q\right) < x^*\left(\underline{\theta}, \beta; p\right)$ and $\lambda'(\theta) \geq 0$ for all $\theta \in \left[\theta_1, \overline{\theta}\right]$; or b) $x^*\left(\overline{\theta}, \beta; q\right) > x^*\left(\underline{\theta}, \beta; p\right)$ and $\widehat{\Upsilon} < 0$, where $\widehat{\Upsilon}$ is given by (91). If these conditions are met, $x^{\min} = x^*\left(\underline{\theta}, \beta; q\right) = 0$ and $V'(0) \leq 0$ implies that it is optimal for the consumer to choose a zero quota, $y = 0$.

Comparative statics

Next we study how the size of the quota y depends on the degree of self-control problems. Assume that an interior solution exists, with $V''(y) < 0$, by the second-order condition. Some straightforward algebra shows that

$$\frac{\partial y}{\partial \beta} = 2\left[f(\theta_2) - f(\theta_1)\right](1 - \beta) \frac{\left[\delta h'(y)\right]^2}{v'(y) V''(y)} \tag{92}$$

In particular, if $f(\theta_2) - f(\theta_1) < 0$, we have $\frac{\partial y}{\partial \beta} > 0$, and more rational consumers choose a higher quota. The condition $f(\theta_2) - f(\theta_1) < 0$ holds, if i) $\theta_2 > \overline{\theta}$ and $\theta_1 \in [\underline{\theta}, \overline{\theta})$; this is essentially the case analyzed in Section 3.1 and Appendix B1. The condition $f(\theta_2) - f(\theta_1) < 0$ also holds ii) if $\theta_1 \in [\underline{\theta}, \overline{\theta})$, $\theta_2 \in [\underline{\theta}, \overline{\theta})$ and $f'(\theta) < 0$.

Marginal sin licenses

Finally, we study a system of *marginal* sin licenses, such that $\tau_2 = \tau_1 + d\tau$ and $q = p + d\tau$, where $d\tau$ is (very) small. The following results will be needed in the proof of Proposition B 1, see Appendix B5. With marginal sin licenses, the first-order condition (87) takes the form

$$\theta_1 v'(y) - \delta h'(y) - p + \frac{1}{\lambda(\theta_1)} v'(y) = 0 \tag{93}$$

The critical value θ_1 is still given by (6), while the second critical value θ_2 is characterized by

$$d\theta_{21} \equiv \theta_2 - \theta_1 = \frac{d\tau}{v'(y)} \tag{94}$$

Combining the first-order condition (93) with equation (6) yields

$$(1-\beta)\,\delta h'(y) - \frac{1}{\lambda(\theta_1)} v'(y) = 0 \tag{95}$$

B3. Proof of Proposition 1: Intermediate steps and further results

a) Interpreting conditions (45) and (46)

Here we show that the optimality conditions (45) and (46) derived in the proof of Propostion 1 is equivalent to the first-order condition (8) characterizing the choice of sin licenses. Notice that the following analysis also applies to the case with $\lambda'(\theta) = 0$: the first-order condition (33) (appearing in the proof of Proposition 1) evidently implies (45).

Let us begin with the case where $\theta_1 > \underline{\theta}$. Remember that $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$. The condition (45) can be re-expressed as

$$v'(y)\left[\theta_2 - \theta_1 - \int_{\theta_1}^{\theta_2} F(\theta)\, d\theta\right] - (1-\beta)\,\delta h'(y)\,(F(\theta_2) - F(\theta_1)) = 0 \tag{96}$$

Next using $-\int_{\theta_1}^{\theta_2} F(\theta)\, d\theta = \int_{\theta_1}^{\theta_2} \theta f(\theta)\, d\theta - \theta_2 F(\theta_2) + \theta_1 F(\theta_1)$, and noting that

$$\int_{\theta_1}^{\theta_2} \theta f(\theta)\, d\theta\, (F(\theta_2) - F(\theta_1))^{-1} = E[\theta \mid \theta_1 \leq \theta \leq \theta_2]$$

allows us to express (96) as

$$\begin{aligned}(F(\theta_2) - F(\theta_1))\left\{\{E[\theta \mid \theta_1 \leq \theta \leq \theta_2] - \theta_1\} v'(y) - (1-\beta)\,\delta h'(y)\right\} \\ + (1 - F(\theta_2))(\theta_2 - \theta_1)\, v'(y) = 0\end{aligned} \tag{97}$$

Finally, from (6) we get $\beta\delta h'(y) - \theta_1 v'(y) = -p$ while (6) and (7) together yield $(\theta_2 - \theta_1)\, v'(y) = q - p$. Plugging these results into (97) gives

$$[F(\theta_2) - F(\theta_1)]\left[E[\theta \mid \theta_1 \leq \theta \leq \theta_2]\, v'(y) - \delta h'(y) - p\right] + [1 - F(\theta_2)](q - p) = 0$$

But this is just the first-order condition (8) characterizing the choice of sin licenses.

Next we analyze the case where $\theta_1 \leq \underline{\theta}$. The condition (46) can be re-expressed as

$$\begin{aligned} & v'(y)\left[\theta_2 - \underline{\theta} - \int_{\underline{\theta}}^{\theta_2} F(\theta)\, d\theta\right] - (1-\beta)\,\delta h'(y)\, F(\theta_2) \\ = \; & \underline{\theta} v'(y) - \beta\delta h'(y) - p \end{aligned} \tag{98}$$

Next using $-\int_{\underline{\theta}}^{\theta_2} F(\theta)\,d\theta = \int_{\underline{\theta}}^{\theta_2} \theta f(\theta)\,d\theta - \theta_2 F(\theta_2)$, and noting that

$$\int_{\underline{\theta}}^{\theta_2} \theta f(\theta)\,d\theta / F(\theta_2) = E[\theta \mid \theta \leq \theta_2]$$

allows us to express (98) as

$$\begin{aligned} &F(\theta_2)\{E[\theta \mid \theta \leq \theta_2]\,v'(y) - (1-\beta)\,\delta h'(y)\} \\ &+ [1 - F(\theta_2)]\,\theta_2 v'(y) = -\beta\delta h'(y) - p \end{aligned} \tag{99}$$

Finally, from (7) we get $\theta_2 v'(y) = \beta\delta h'(y) + q$. Plugging this result into (99) yields

$$F(\theta_2)\left[E[\theta \mid \theta \leq \theta_2]\,v'(y) - \delta h'(y) - p\right] + [1 - F(\theta_2)]\,(q - p) = 0$$

But this is just the first-order condition (8) characterizing the choice of sin licenses, when $\theta_1 \leq \underline{\theta}$.

b) Analyzing the case where the condition $\lambda'(\theta) \geq 0$ does not hold for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Here we show that the allocation $x(\theta)$ chosen by the consumer may be implementable with sin licenses, even when the condition $\lambda'(\theta) \geq 0$ does not hold for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Assumptions and some preliminary findings

We adopt the following assumptions:

a) Hazard rate. We assume that $\lambda'(\theta) > 0$ for $\theta < \theta_L^{*IC}$ and $\theta > \theta_H^{IC}$; $\lambda'(\theta) \leq 0$ for $\theta \in \left[\theta_L^{*IC}, \theta_H^{IC}\right]$, where $0 \leq \theta_L^{*IC} < \theta_H^{IC} \leq \overline{\theta}$, and strict inequality $\lambda'(\theta) < 0$ holds for a subset of positive measure. (Here the superscript IC refers to the second-order incentive constraint. Below, it will become clear, why there is an asterisk in θ_L^{*IC}.)

Motivation for these assumptions:

i) Let us first assume that $\overline{\theta}$ is finite. Then $\lim_{\theta\to\overline{\theta}} \lambda(\theta) = \lim_{\theta\to\overline{\theta}} f(\theta)/[1 - F(\theta)] = \infty$ if $f\left(\overline{\theta}\right) > 0$, while $\lim_{\theta\to\overline{\theta}} \lambda(\theta) = \lim_{\theta\to\overline{\theta}} -f'(\theta)/f(\theta) > \infty$ if $f\left(\overline{\theta}\right) = 0$ and $f(\theta) > 0$ for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$. Since $\lambda(\theta)$ goes to infinity when $\theta \to \overline{\theta}$, the hazard rate is increasing at the right end of the distribution. Hence if $\overline{\theta}$ is finite, $\lambda'(\theta) > 0$ for $\theta > \theta_H^{IC}$. Notice, however, that we also allow for the case, where $\lambda'(\theta) < 0$ at the right end of the distribution. This is possible, if the support of the distribution has no finite upper bound, and $\theta_H^{IC} = \overline{\theta} = \infty$.

ii) Assume that the distribution of θ is single-peaked, and the peak $\theta^{\max} > \underline{\theta}$. Then $f'(\theta) > 0$ at the left end of the distribution, and consequently $\lambda'(\theta) = [f'(\theta)/f(\theta) + \lambda(\theta)]\,\lambda(\theta) > 0$ for $\theta < \theta_L^{*IC}$. Notice, however, that we also allow for the case where the peak $\theta^{\max} = \underline{\theta}$. Then it is possible that $\lambda'(\theta) < 0$ also at the left end of the distribution; in this case we set $\theta_L^{*IC} < \underline{\theta}$.

iii) Since there are good reasons to argue that the hazard rate should be increasing at both ends of the distribution (items i and ii), the most natural part of the distribution where the hazard rate can be decreasing is the middle part. Hence $\lambda'(\theta) \leq 0$ for $\theta \in \left[\theta_L^{IC}, \theta_H^{IC}\right]$, where strict inequality $\lambda'(\theta) < 0$ holds for a subset of positive measure. Notice that we also allow for the possibility that $\lambda'(\theta) < 0$ for all θ. In this situation $\theta_L^{*IC} < \underline{\theta}$ and $\theta_H^{IC} = \overline{\theta} = \infty$.

Next we analyze in which parts of the state space $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ the second-order incentive constraint (19) prevents the the unconstrained solution from being implemented. To do so, we also need to take into account the constraint at lower bound $\underline{\theta}$. To do so, we define a threshold value θ_L^{**IC} as follows: $\theta_L^{**IC} = \underline{\theta}$ if $x^u\left(\underline{\theta}^+\right) < x^u\left(\underline{\theta}\right)$ - ie. $x^u\left(\underline{\theta}\right)$ cannot be implemented - , and $\theta_L^{**IC} < \underline{\theta}$ otherwise (see (38)). We then combine the constraints deriving from the hazard rate, and from pricing at the lower bound: more formally, let $\theta_L^{IC} = \max\left\{\theta_L^{*IC}, \theta_L^{**IC}\right\}$. Now we can state the following results: In the subset

$$\Theta^{IC} = [\underline{\theta}, \theta_L^{IC}] \cup [\theta_H^{IC}, \overline{\theta}]$$

the second-order incentive constraint (19) prevents the unconstrained $x^u(\theta)$ solution from being implemented. Notice in particular, that Θ^{IC} may be non-empty even in the special case, where $\lambda'(\theta) < 0$ for all θ (that is $\theta_L^{*IC} < \underline{\theta}$ and $\theta_H^{IC} = \overline{\theta} = \infty$); this is possible if the $x^u\left(\underline{\theta}^+\right) < x^u\left(\underline{\theta}\right)$ and the unconstrained solution does not satisfy the second-order incentive constraint (19) at the lower bound $\underline{\theta}$; see (38). Meanwhile, in the subset

$$\Theta^{NIC} = \left(\theta_L^{IC}, \theta_H^{IC}\right)$$

the second-order incentive constraint (19) does not prevent $x^u(\theta)$ from being implemented; moreover $dx^u(\theta)/d\theta > 0$ in a subset of Θ^{NIC} with positive measure.

b) We assume that the slope of the pricing scheme that implements the unconstrained solution $T'(x^u(\theta))$ is increasing over $\left[\underline{\theta}, \overline{\theta}\right]$. A sufficient condition is that $d[\theta\lambda(\theta)]/d\theta = \lambda(\theta) + \theta\lambda'(\theta) > 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. We adopt this assumption in order to keep the analysis relatively simple and clear: Given this assumption, there is at most one solution to the equation $T'(x^u(\theta)) = p$ over the interval $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Let us denote this solution by θ^p. If $T'(x^u(\theta)) > p$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, we set $\theta^p = \underline{\theta}$; if $T'(x^u(\theta)) < p$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ we set we set $\theta^p = \overline{\theta}$. Likewise, there is at most one solution to the equation $T'(x^u(\theta)) = q$ over the interval $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Let us denote this solution by θ^q. If $T'(x^u(\theta)) > q$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, we set $\theta^q = \underline{\theta}$; if $T'(x^u(\theta)) < q$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ we set we set $\theta^q = \overline{\theta}$.

Then in the subset

$$\Theta^{PC} = [0, \theta^p] \cup [\theta^p, \infty]$$

the unconstrained solution $x^u(\theta)$ cannot be implemented due to a pricing constraint (either

(12) or (13)). However, in the subset

$$\Theta^{NPC} = (\theta^p, \theta^q)$$

the unconstrained solution is not ruled out by a pricing constraint.

c) Combining results from a) and b) we further define the following sets yields the following additional findings:

In the set

$$\Theta^{C} = \Theta^{IC} \cup \Theta^{PC}$$

the unconstrained solution $x^u(\theta)$ is ruled out either by the second-order incentive constraint (19), or by a pricing constraint - either (12) or (13).

In the set

$$\Theta^{NC} = \Theta^{NIC} \cap \Theta^{NPC}$$

the unconstrained solution is not ruled out by any or the constraints ((19), (12) or (13)); notice, however, that while the ex ante consumer can implement $x^u(\theta)$ for $\theta \in \Theta^{NC}$, he does not necessarily want to do that.

In the set

$$\Theta^{IC\&NPC} = \Theta^{IC} \cap \Theta^{NPC}$$

the unconstrained solution is ruled out by the second-order incentive constraint (19), but it is not ruled out by the a pricing constraint.

In particular, the subsets Θ^{NC} and $\Theta^{IC\&NPC}$ play a key role in what follows.

Results

a) If $\Theta^{NC} = \emptyset$, the mechanism chosen by the consumer can be implemented with sin licenses.. Proof: In this situation, one of the constraints (19), (12) and (13) necessarily binds for any $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Then the analysis presented in Appendix A1, Step 4, can be directly applied.

b) If $\Theta^{NC} \neq \emptyset$ and $\Theta^{IC\&NPC} = \emptyset$, the allocation chosen by the consumer cannot be implemented with sin licenses. Proof: $\Theta^{NC} \neq \emptyset$ and $\Theta^{IC\&NPC} = \emptyset$ is equivalent to $\Theta^{NC} = \Theta^{NPC} \subseteq \Theta^{NIC}$. Then it is easy to see that the consumer implements the following allocation: i) $x(\theta) = x^u(\theta)$ for $\theta \in \Theta^{NC} = \Theta^{NPC} = [\theta^p, \theta^q]$; the unconstrained solution $x^u(\theta)$ maximizes the ex ante consumer's objective function J, with a given shock realization θ. ii) $x(\theta) = x^*(\theta, \beta; p)$ for $\theta < \theta^p$; here the consumer would like to choose $x(\theta) > x^*(\theta, \beta; p)$ and $T'(x(\theta)) < p$, but this is infeasible due to the pricing constraint (12). iii) $x(\theta) = x^*(\theta, \beta; q)$ for $\theta > \theta^q$; here the consumer would like to choose $x(\theta) < x^*(\theta, \beta; q)$

and $T'(x(\theta)) > q$, but this is infeasible due to the pricing constraint (13). Notice that the second-order incentive constraint (19) does not bind for any θ, and there is no bunching in equilibrium. Also notice, that over the interval $\theta \in \Theta^{NPC} = [\theta^p, \theta^q]$, the allocation $x^u(\theta)$ is implemented with the non-linear pricing schedule $T'(x^u(\theta))$, given by (34). Hence, the allocation $x(\theta)$ chosen by the consumer cannot be implemented with sin licenses.

c) If $\Theta^{NC} \neq \emptyset$ and $\Theta^{IC\&NPC} \neq \emptyset$, the allocation chosen by the consumer may or may not be implementable with sin licenses. Let us consider a change in price floor p and/or the price ceiling q. If and only if the set Θ^{NC} becomes smaller, or the set $\Theta^{IC\&NPC}$ becomes larger, it is more likely that that the allocation chosen by the consumer can be implemented with sin licenses.

This situation requires some more detailed analysis. In particular, in this situation we have a non-empty set $\Theta^{IC\&NPC} = \Theta^{IC} \cap \Theta^{NPC} \neq \emptyset$, where the equilibrium allocation will involve bunching, since the the second-order incentive constraint (19) will bind in equilibrium. On the other hand, there is also a non-empty set $\Theta^{NC} = \Theta^{NIC} \cap \Theta^{NPC} \neq \emptyset$, where the unconstrained solution $x^u(\theta)$ could be in principle implemented. However, the consumer may choose not to implement $x^u(\theta)$ for (some) $\theta \in \Theta^{NC}$ because this would violate incentive constraints for (some) $\theta \in \Theta^{IC\&NPC}$. To illustrate, consider a candidate solution with $x(\theta) = y$ for $\theta \in \Theta^{IC\&NPC}$ and $x(\theta) = x^u(\theta)$ for $\theta \in \Theta^{NC}$. Further assume that $x(\theta') = x^u(\theta') > y = x(\theta'')$ for some $\theta' \in \Theta^{NC}$, $\theta'' \in \Theta^{IC\&NPC}$ such that $\theta' < \theta''$. Clearly, this allocation cannot be implemented, since ex post type θ'' would choose the allocation intended for θ'. Then to render $x(\theta) = y$ implementable in $\Theta^{IC\&NPC}$, the ex ante consumer may choose to implement $x(\theta) = y$,rather than $x^u(\theta)$, also in Θ^{NC}. Finally, if the equilibrium allocation is such that for each θ, we have either $x(\theta) = y$, $x(\theta) = x^*(\theta, \beta; p)$ or $x(\theta) = x^*(\theta, \beta; q)$, we can conclude that the allocation is implementable with sin licenses.

To find out whether or not the allocation $x(\theta)$ chosen by the ex ante consumer is implementable with sin licenses, we proceed as follows. First, we establish a candidate solution, using the same steps as in Appendix A1. Hence, we maximize the Lagrangian (39), subject to (12), (13) and (19), and get the optimality conditions (40), (41), (42), (43) and (44). Following the same procedure as in Appendix A1, we can further conclude that the candidate solution (y, θ_1, θ_2) is characterized by the equations (6), (7) and (45), if $\theta_1 > \underline{\theta}$ and by (6), (7) and (46), if $\theta_1 \leq \underline{\theta}$.

Second, we check whether or not the candidate solution is the (constrained) optimal solution for the ex ante consumer. If both $\theta_1 \in \Theta^C$ and $\theta_2 \in \Theta^C$, the consumer can do no better and the candidate solution is the optimal solution that the consumer chooses. The equilibrium allocation $x(\theta)$ can be implemented with sin licenses. On the other hand, if either $\theta_1 \in \Theta^{NC}$ or $\theta_2 \in \Theta^{NC}$ (or maybe both θ_1 and θ_2 belong to Θ^{NC}), the consumer can improve upon the candidate solution. Then the equilibrium allocation $x(\theta)$ will also involve the unconstrained solution $x^u(\theta)$ (for some subinterval in the type space $\theta \in [\underline{\theta}, \overline{\theta}]$), and the allocation the consumer chooses cannot be implemented with sin licenses. Also, in such a situation, (at least) one of the boundaries will be determined by the condition

$$(1-\beta)\,\delta h'(y) - \frac{1}{\lambda(\theta_i)} v'(y) = 0, \tag{100}$$

where $i = 1$, if θ_1 is determined by (100), while $i = 2$, if θ_2 is determined by (100).

Next, we turn to comparative statics and study how changes in p and q affect the implementability through sin licenses.. We want to show that it is more likely that the allocation chosen by the consumer can be implemented with sin licenses, if and only if the set Θ^{NC} becomes smaller, or the set $\Theta^{IC\&NPC}$ becomes larger.

i) Let us first consider a case where $\theta_L^{IC} < \theta^p < \theta_H^{IC} < \theta^q$. Here $\Theta^{NC} = \left(\theta^p, \theta_H^{IC}\right)$ and $\Theta^{IC\&NPC} = \left[\theta_H^{IC}, \theta^q\right]$. Hence in this situation an increase in p, which renders θ^p larger and the set Θ^{NC} smaller, makes it more likely that the equilibrium allocation can be implemented with sin licenses. Likewise an increase in q, which renders θ^q larger so that also the set $\Theta^{IC\&NPC}$ becomes larger, makes it more likely that the equilibrium allocation can be implemented with sin licenses.

To demonstrate these properties, let us assume that the allocation the consumer wants to implement includes the unconstrained solution, over a subinterval in the type (θ) space. More formally, let $x(\theta) = x^*(\theta, \beta; p)$ for $\theta \in \Theta^p = [\underline{\theta}, \theta^p]$, $x(\theta) = x^u(\theta)$ for $\theta \in \Theta^u = (\theta^p, \theta_1)$, $x(\theta) = y$ for $\theta \in \Theta^y = [\theta_1, \theta_2]$ and $x(\theta) = x^*(\theta, \beta; q)$ for $\theta \in \Theta^q = (\theta_2, \overline{\theta}]$. Here $y,\ \theta_1$ and θ_2 are determined by (45) and (7), together with the equation

$$(1-\beta)\,\delta h'(y) - \frac{1}{\lambda(\theta_1)} v'(y) = 0, \tag{101}$$

which states that the unconstrained solution is valid at θ_1, or $x^u(\theta_1) = y$. For the solution to be of this type, we must have $\theta_1 \in \Theta^{NC}$ (since $x^u(\theta_1)$ must be feasible) and $\theta_2 > \theta_H^{IC}$ (for the equation (45) to hold, the interval $\Theta^y = (\theta_1, \theta_2)$ with bunching $x(\theta) = y$ must contain a subinterval where $\lambda'(\theta) > 0$).

Next, let us consider a limiting case, where $\theta_1 = \theta^p$. Hence the set where the unconstrained solution $x^u(\theta)$ is implemented is a single point $\Theta^u = \theta_1 = \theta^p$. Next let us examine what happens to the set Θ^u when we change p and/or q. In particular if the set Θ^u vanishes (we get $\theta_1 < \theta^p$, implying that $\Theta^u = \emptyset$), the allocation chosen by the consumer includes only segments $x(\theta) = x^*(\theta, \beta; p)$, $\ x(\theta) = y$ and $x(\theta) = x^*(\theta, \beta; q)$, and the allocation can be implemented with sin licenses.

Studying the effects of a change in p is easy. Since p does not appear in any of the equations (45), (7) and (101), the threshold value θ_1 does not change when p is altered. On the other hand, remember that θ^p is determined by $T^{u\prime}(x^u(\theta^p)) = p$. We then get $\frac{d\theta^p}{dp} = \left[d\left[T^{u\prime}(x^u(\theta^p))\right]/d\theta\right]^{-1} > 0$; remember that we assume that $T^{u\prime}(x^u(\theta))$ is increasing in θ. Hence we can conclude that if p is increased, we end up in a situation where $\theta^p > \theta_1$ and $\Theta^u = \emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

Next, a change in q affects the threshold value θ_1, while leaving θ^p intact. Totally differentiating (45), (7) and (101) one can show that

$$\begin{aligned}\frac{d\theta_1}{dq} &= \left[\frac{\frac{1}{\lambda(\theta_1)}v''(y)-(1-\beta)\delta h''(y)}{\lambda'(\theta_1)\frac{v'(y)}{[\lambda(\theta_1)]^2}}\right]\times \\ &\left[\frac{\left\{\int_{\theta_1}^{\theta_2}\left[\frac{1}{\lambda(\theta)}v''(y)-(1-\beta)\delta h''(y)\right]f(\theta)\,d\theta\right\}}{\left[(1-\beta)\delta h'(y)-\frac{1}{\lambda(\theta_2)}v'(y)\right]f(\theta_2)}v'(y)+\left[\theta_2 v''(y)-\beta\delta h''(y)\right]\right]^{-1} \\ &< 0\end{aligned} \tag{102}$$

When signing this expression, we have used the following observations: i) $\lambda'(\theta_1)<0$ and ii) $(1-\beta)\delta h'(y)-\frac{1}{\lambda(\theta_2)}v'(y)>0$. The second inequality arises, since $\int_{\theta_1}^{\theta_2}\left[(1-\beta)\delta h'(y)-\frac{1}{\lambda(\theta)}v'(y)\right]f(\theta)=0$, $(1-\beta)\delta h'(y)-\frac{1}{\lambda(\theta_1)}v'(y)=0$, $\lambda'(\theta)\leq 0$ for $\theta\in[\theta_1,\theta_H^{IC})$ and $\lambda'(\theta)>0$ for $\theta\in[\theta_H^{IC},\theta_2]$. Hence, we can conclude that if q is increased, we end up in a situation where $\theta_1<\theta^p$ and $\Theta^u=\emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

ii) Let us next examine the case where $\theta^p<\theta_L^{IC}<\theta^q<\theta_H^{IC}$. Here $\Theta^{NC}=\left(\theta_L^{IC},\theta^q\right)$ and $\Theta^{IC\&NPC}=\left[\theta^p,\theta_L^{IC}\right]$. Hence in this situation a *decrease* in p, which renders θ^p smaller and the set $\Theta^{IC\&NPC}$ larger, makes it more likely that the equilibrium allocation can be implemented with sin licenses. On the other hand, a *decrease* in q, which renders θ^q smaller so that the set Θ^{NC} becomes smaller, makes it more likely that the equilibrium allocation can be implemented with sin licenses.

Notice in that a special case here is the situation where $\lambda'(\theta)<0$ for all θ, but the unconstrained solution $x^u(\theta)$ does not satisfy the second-order incentive constraint (19) at the lower bound. In this setup, $\Theta^{NC}=(\underline{\theta},\theta^q)$ and $\Theta^{IC\&NPC}=\underline{\theta}$.

We analyze this situation more formally, by following essentially the same steps as above, in case i). The only small extra twist is that we have to take into consideration the possibility that $\theta_1\leq\underline{\theta}$. Hence, let us assume that the allocation the consumer wants to implement includes the unconstrained solution, over a subinterval in the type (θ) space. More formally, let $x(\theta)=x^*(\theta,\beta;p)$ for $\theta\in\Theta^p=[\underline{\theta},\theta_1)$, $x(\theta)=y$ for $\theta\in\Theta^y=[\theta_1,\theta_2]$, $x(\theta)=x^u(\theta)$ for $\theta\in\Theta^u=(\theta_2,\theta^q)$, and $x(\theta)=x^*(\theta,\beta;q)$ for $\theta\in\Theta^q=\left[\theta^q,\overline{\theta}\right]$. If $\theta_1>\underline{\theta}$, y, θ_1 and θ_2 are determined by (45) and (6), together with the equation

$$(1-\beta)\delta h'(y)-\frac{1}{\lambda(\theta_2)}v'(y)=0, \tag{103}$$

which states that the unconstrained solution is valid at θ_2, or $x^u(\theta_2)=y$. If $\theta_1\leq\underline{\theta}$, y and θ_2 are determined by (46) and (103). For the solution to be of this type, we must have $\theta_2\in\Theta^{NC}$ (since $x^u(\theta_2)$ must be feasible) and either $\theta_1<\theta_L^{IC}$ (for the equation (45) to hold, the interval $\Theta^y = (\theta_1, \theta_2)$ with bunching $x(\theta) = y$ must contain a subinterval where $\lambda'(\theta) > 0$) or $\theta_1 \leq \underline{\theta}$ (this is the situation where the second-order incentive constraint (19) is not satisfied at the lower boundary $\underline{\theta}$).

Next, let us consider a limiting case, where $\theta_2 = \theta^q$. Hence the set where the unconstrained solution $x^u(\theta)$ is implemented is a single point $\Theta^u = \theta_2 = \theta^q$. Next let us examine what happens to the set Θ^u when we change p and/or q. In particular if the set Θ^u vanishes (we get $\theta_2 > \theta^q$, implying that $\Theta^u = \emptyset$), the allocation chosen by the consumer includes only segments $x(\theta) = x^*(\theta, \beta; p)$, $x(\theta) = y$ and $x(\theta) = x^*(\theta, \beta; q)$, and the allocation can be implemented with sin licenses.

Studying the effects of a change in q is easy. Since q does not appear in any of the equations (45), (6) and (103), the threshold value θ_2 does not change when q is altered. On the other hand, remember that θ^q is determined by $T^{u\prime}(x^u(\theta^q)) = q$. We then get $\frac{d\theta^q}{dq} = [d[T^{u\prime}(x^u(\theta^p))]/d\theta]^{-1} > 0$; remember that we assume that $T^{u\prime}(x^u(\theta))$ is increasing in θ. Hence we can conclude that if q is decreased, we end up in a situation where $\theta^q < \theta_2$ and $\Theta^u = \emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

Next, a change in p affects the threshold value θ_2, while leaving θ^q intact. Totally differentiating (45), (6) and (103), if $\theta_1 > \underline{\theta}$, or alternatively (46), and (103), if $\theta_1 \leq \underline{\theta}$, one can show that

$$
\begin{aligned}
\frac{d\theta_2}{dp} &= \left[\frac{\frac{1}{\lambda(\theta_2)} v''(y) - (1-\beta)\delta h''(y)}{\lambda'(\theta_2) \frac{v'(y)}{[\lambda(\theta_2)]^2}} \right] \times \qquad (104) \\
&\left[-\frac{\left\{ \int_{\theta_1}^{\theta_2} \left[\frac{1}{\lambda(\theta)} v''(y) - (1-\beta)\delta h''(y) \right] f(\theta)\, d\theta \right\}}{\left[(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta_1)} v'(y) \right] f(\theta_1)} v'(y) + \left[\theta_1 v''(y) - \beta\delta h''(y) \right] \right]^{-1} \\
&< 0
\end{aligned}
$$

When signing this expression, we have used the following observations: i) $\lambda'(\theta_2) < 0$ and ii) $(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta_2)} v'(y) < 0$. The second inequality arises, since

$$
\int_{\theta_1}^{\theta_2} \left[(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta)} v'(y) \right] f(\theta)\, d\theta = 0,
$$

$(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta_2)} v'(y) = 0$, $\lambda'(\theta) > 0$ (and hence $(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta)} v'(y)$ is increasing in θ) for $\theta \in \left[\theta_1, \theta_L^{IC}\right]$ and $\lambda'(\theta) \leq 0$ (and hence $(1-\beta)\delta h'(y) - \frac{1}{\lambda(\theta)} v'(y)$ is decreasing in θ) for $\theta \in \left(\theta_L^{IC}, \theta_2\right)$. Hence, we can conclude that if p is *decreased*, we end up in a situation where $\theta_2 > \theta^q$ and $\Theta^u = \emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

iii) Finally, we examine the case where $\theta^p < \theta_L^{IC} < \theta_H^{IC} < \theta^q$. Here $\Theta^{NC} = \left(\theta_L^{IC}, \theta_H^{IC}\right)$

while $\Theta^{IC\&NPC} = \left[\theta^p, \theta_L^{IC}\right] \cup \left[\theta_H^{IC}, \theta^q\right]$. Hence in this situation a *decrease* in p, which renders θ^p smaller and the set $\Theta^{IC\&NPC}$ larger, makes it more likely that the equilibrium allocation can be implemented with sin licenses. On the other hand, an *increase* in q, which renders θ^q larger and the set $\Theta^{IC\&NPC}$ larger, makes it more likely that the equilibrium allocation can be implemented with sin licenses.

To demonstrate these properties, let us assume that the allocation the consumer wants to implement includes the unconstrained solution, over a subinterval in the type (θ) space. More formally, let $x(\theta) = x^*(\theta, \beta; p)$ for $\theta \in \Theta^p = [\underline{\theta}, \theta_{1L})$, $x(\theta) = y_L$ for $\theta \in \Theta^{y_L} = [\theta_{1L}, \theta_{2L}]$, $x(\theta) = x^u(\theta)$ for $\theta \in \Theta^u = (\theta_{2L}, \theta_{1H})$, $x(\theta) = y_H$ for $\theta \in \Theta^{yH} = [\theta_{1H}, \theta_{2H}]$ and $x(\theta) = x^*(\theta, \beta; q)$ for $\theta \in \Theta^q = (\theta_{2H}, \overline{\theta}]$. Notice in particular that in this putative solution, there are two subsets with bunching Θ^{y_L} and Θ^{y_H}. Here y_H, θ_{1H} and θ_{2H} are determined by the equations (45), (7) and (101) (where evidently, $y = y_H, \theta_1 = \theta_{1H}$ and $\theta_2 = \theta_{2H}$). On the other hand, y_L, θ_{1L} and θ_{2L} are determined by (45), (6) and (103), if $\theta_{1L} > \underline{\theta}$, while y_L, θ_{1L} and θ_{2L} are determined by (46), (6) and (103) if $\theta_{1L} = \underline{\theta}$ (here evidently, $y = y_L, \theta_1 = \theta_{1L}$ and $\theta_2 = \theta_{2L}$).

Next, let us consider a limiting case, where $\theta_{2L} = \theta_{1U}$. Hence the set where the unconstrained solution $x^u(\theta)$ is implemented is a single point $\Theta^u = \theta_{2L} = \theta_{1H}$. Next let us examine what happens to the set Θ^u when we change p and/or q. In particular if the set Θ^u vanishes (we get $\theta_{2L} > \theta_{1U}$, implying that $\Theta^u = \emptyset$), the allocation chosen by the consumer includes only segments $x(\theta) = x^*(\theta, \beta; p)$, $x(\theta) = y_L = y_H = y$ and $x(\theta) = x^*(\theta, \beta; q)$, and the allocation can be implemented with sin licenses.

From the discussion above, it should be clear that θ_{2L} depends on p, while it does not depend on q. On the other hand, θ_{1H} depends on q, while it does not depend on p.

Now, totally differentiating (45), (6) and (103), if $\theta_{1H} > \underline{\theta}$, or alternatively (46), and (103), if $\theta_{1H} \leq \underline{\theta}$, one can show that $d\theta_{2L}/dp$ is given by (104). Hence, we can conclude that if p is *decreased*, we end up in a situation where $\theta_{2L} > \theta_{1H}$ and $\Theta^u = \emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

Next, totally differentiating (45), (7) and (101) one can show that $d\theta_{1H}/dq$ is given by (102). Hence, we can conclude that if q is *increased*, we end up in a situation where $\theta_{2L} > \theta_{1H}$ and $\Theta^u = \emptyset$. Then the allocation chosen by the ex ante consumer can be implemented with sin licenses.

B4. Sin taxes vs. sin licenses: the role of the distribution of self-control problems

In this section of the Appendix, we analyze how the welfare properties of sin licenses, *vis-à-vis* linear sin taxes, depend on the distribution of self-control problems β. The main, rather intuitive, argument we try to make is the following: Personalized regulation, in the form of sin licenses, is more likely to improve welfare, if consumers differ significantly from each other in terms of self-control problems.

To conduct the analysis, we need to introduce some more structure. Assume that the population consists of continuum of different groups I. Consumers belonging to the same group I share the same utility function $v_I(x)$, the same harm function $h_I(x)$, the same discount factor δ_I, and the same cumulative distribution function of preference shocks $F_I(\theta)$. Consumers within a group I may differ in terms of self-control problems (β); in group I, β has a cumulative distribution function $M_I(\beta)$. Finally notice that the structure introduced here is quite general and flexible: consumers belonging to different groups, say I and I', may differ from each other with respect to all the above-listed characteristics.

Some useful definitions: Let us consider consumers belonging to a certain group I. A consumer's *ex ante type* is given by the consumer's group I, together with his degree of self-control problems β. The consumer's *ex post type* is given by I and β together with the consumer's preference shock realization θ.

Assumptions concerning sin licenses. Throughout this appendix, we assume that $p = s$ and $q > s$ (where p is the price floor and q is the price ceiling under the system of sin license, while $s = 1 + \tau$ is the consumer price when a universal sin tax τ is in place). That is, the consumers can only choose personalized regulation schemes that are more stringent than the universal tax-based scheme.

Auxiliary results and assumptions. In the analysis below, we need some auxiliary results. Remember that realized consumption $x(\theta, \beta, r)$ is given by the equation $\theta v'(x) - \beta\delta h'(x) - r = 0$, where $r \in \{s, p, q\}$. Then we have

$$\frac{\partial x}{\partial r} = \frac{1}{\theta v''(x) - \beta\delta h''(x)} < 0 \tag{105}$$

$$\frac{\partial x}{\partial \beta} = \frac{\delta h'(x)}{\theta v''(x) - \beta\delta h''(x)} = \delta h'(x)\frac{\partial x}{\partial r} < 0 \tag{106}$$

Also

$$\frac{\partial^2 x}{\partial r \partial \beta} = \left[\frac{h''(x)}{h'(x)} - \left(\frac{\theta v'''(x) - \beta\delta h'''(x)}{\theta v''(x) - \beta\delta h''(x)}\right)\right]\left(\frac{\partial x}{\partial \beta}\right)\left(\frac{\partial x}{\partial r}\right) \tag{107}$$

and

$$\begin{aligned}\frac{\partial^2 x}{\partial \beta^2} &= \left[2\left(\frac{h''(x)}{h'(x)}\right) - \left(\frac{\theta v'''(x) - \beta\delta h'''(x)}{\theta v''(x) - \beta\delta h''(x)}\right)\right]\left(\frac{\partial x}{\partial \beta}\right)^2 \\ &= \left(\frac{\partial x}{\partial \beta}/\frac{\partial x}{\partial r}\right)\frac{\partial^2 x}{\partial r \partial \beta} + \left(\frac{h''(x)}{h'(x)}\right)\left(\frac{\partial x}{\partial \beta}\right)^2\end{aligned} \tag{108}$$

Following Haavio and Kotakorpi (2011), we further assume that

$$\frac{\partial^2 x}{\partial r \partial \beta} \geq 0 \tag{109}$$

$$\frac{\partial^2 x}{\partial \beta^2} \geq 0 \tag{110}$$

Using expression (108), one can clearly see that (109) implies (110). Hence in what follows we focus on assumption (109). Assumption (109) essentially means that the demand of irrational consumers with a high level of consumption is more responsive (in absolute terms) to price changes than the demand of rational consumers with a low or moderate level of consumption. A basic rationale for this feature is that as rational consumers consume relatively little of harmful goods in any case, higher prices cannot reduce their consumption much further. It is important to note that the condition concerns *absolute* changes in demand. Even with this assumption, demand can be less elastic for heavy users than for moderate consumers. Using (107), one can show that the inequality (109) holds if i) $v'''(x) \geq 0$ and ii) $h(x)$ is either linear or $\frac{h'''(x)h'(x)}{[h''(x)]} \leq 1$ (sufficient conditions).The conditions i) and ii) are satisfied for commonly used functional forms, for example when v is of the CRRA or CARA-variety, or quadratic, and when the harm function is exponential or $h(x) = x^s$ where $s \geq 1$. The assumption (109) is further discussed and motivated in Haavio and Kotakorpi (2011).

In addition, we need one further assumption concerning the density function of the preference shock: we assume that

$$f_I'(\theta) \leq 0. \tag{111}$$

The distribution of self-control problems and private benefits from introducing sin licenses

As a first result, we show that private benefits (ΔV) from introducing sin licenses are larger when consumers differ more in terms of self-control problems. To be more specific, we consider two distributions of self-control problems, $M_I(\beta)$ and $\widehat{M_I}(\beta)$. Both distributions have the same mean but $\widehat{M_I}(\beta)$ is more dispersed, in the sense of second-order stochastic dominance. We show that, in each group I, the private benefits from the reform (ΔV) are larger under the more dispersed distribution $\widehat{M_I}(\beta)$.

How to apply the result? Ultimately, we are interested in the impact of the reform on social welfare (ΔW). However, since the change in tax revenues (ΔTR) is readily observable, a result that allows us to evaluate the size of ΔV also enables us to better evaluate $\Delta W = \Delta V + \Delta TR$. Hence we get a two-step procedure to evaluate the reform. First, if tax revenues do not decrease ($\Delta TR \geq 0$) when sin licenses are introduced, we can conclude that the reform has been welfare improving ($\Delta W > 0$). Second, if the first criterion is met, and if we furthermore know (or have reasons to believe) that consumers differ significantly in terms of self-control problems, the reform can bring about significant welfare gains (i.e. ΔW can be large).

Proving the result. To prove the result concerning private benefits (ΔV), we need the assumptions (109), (110) and (111) given above. In addition, following Haavio and Kotakorpi

(2011, Section 4.2; see in particular Proposition 6) we assume that

$$\frac{\partial}{\partial\beta}\left[(1-\beta)\frac{\partial^2 x}{\partial\beta^2}\right]\leq 0 \tag{112}$$

The condition that $(1-\beta)\frac{\partial^2 x(q;\beta)}{\partial\beta^2}$ should be non-increasing in β holds for many commonly used functional forms, for example when v is of the CRRA or CARA-variety or quadratic, and when the harm function is linear or $h(x)=x^s$ where $s\geq 2$. Further, in order to interpret the condition that $(1-\beta)\frac{\partial^2 x(q;\beta)}{\partial\beta^2}$ should be non-increasing in β, Haavio and Kotakorpi (2011) show that this holds (approximately) if a price change affects the health of irrational consumers (heavy users) more than the health of rational consumers.

A) We begin the analysis by considering the impact of the reform on the private welfare measure of a consumer with ex ante type (I,β)

$$\Delta V_I(\beta)=V_{\ell I}(\beta;y,p,q)-V_{\tau I}(\beta;s)$$

where $V_{\ell I}(\beta;y,p,q)$ is the consumer's ex ante welfare measure under sin licenses and $V_{\tau I}(\beta;s)$ is the consumer's ex ante welfare measure under sin taxes. Now, our key task is to show that $\Delta V_I(\beta)$ is a convex function of β; after this is done, the result we want to prove follows almost immediately.

Straightforward differentiation shows that

$$\begin{aligned}\frac{d^2[\Delta V(\beta)]}{d\beta^2} &= \frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial\beta^2}-\frac{\partial^2\ V_\tau(\beta;s)}{\partial\beta^2}+2\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y\partial\beta}\frac{dy}{d\beta}\\ &\quad+\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y^2}\left(\frac{dy}{d\beta}\right)^2+\frac{\partial V_\ell(y;\beta;p,q)}{\partial y}\frac{d^2 y}{d\beta^2}\end{aligned} \tag{113}$$

Next, notice the following points: i) Since y is chosen optimally by the consumer, $\frac{\partial V_\ell(y;\beta;p,q)}{\partial y}=0$. ii) $\frac{dy}{d\beta}=-\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y\partial\beta}/\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y^2}$. Then the expression (113) simplifies to

$$\frac{d^2[\Delta V(\beta)]}{d\beta^2}=\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial\beta^2}-\frac{\partial^2 V_\tau(\beta;s)}{\partial\beta^2}-\frac{\left(\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y\partial\beta}\right)^2}{\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y^2}} \tag{114}$$

The last term $-\left(\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y\partial\beta}\right)^2/\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y^2}>0$, if the second-order condition $\left(\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial y^2}<0\right)$ holds (i.e. if there is an interior solution to the consumer's license choice problem). Hence, to sign the expression (114), we still need to analyze the term $\frac{\partial^2 V_\ell(y;\beta;p,q)}{\partial\beta^2}-\frac{\partial^2 V_\tau(\beta;s)}{\partial\beta^2}$.

To carry out the analysis, remember that we consider here licenses schemes with the properties $p=s$ and $q>s$. This assumption implies that, for shock realizations $\theta\leq\theta_1$, the consumers chooses same level of consumption $x^*(\theta,\beta;s)$ under both regulatory schemes (sin taxes and sin licenses). Then

$$\Delta V(\beta) = \int_{\theta_1}^{\theta_2} \Psi_1(y, \theta, \beta, s)\, dF(\theta) + \int_{\theta_2}^{\overline{\theta}} \Psi_2(y, \theta, \beta, s, q)\, dF(\theta)$$

where

$$\Psi_1(y, \theta, \beta, s) = [\theta v(y) - \delta h(y) - sy] - [\theta v(x^*(\theta, \beta; s)) - \delta h(x^*(\theta, \beta; s)) - sx^*(\theta, \beta; s)]$$

and

$$\begin{aligned} \Psi_2(y, \theta, \beta, s, q) &= [\theta v(x^*(\theta, \beta; q)) - \delta h(x^*(\theta, \beta; q)) - q(x^*(\theta, \beta; q) - y) - sy] \\ &\quad - [\theta v(x^*(\theta, \beta; s)) - \delta h(x^*(\theta, \beta; s)) - sx^*(\theta, \beta; s)] \end{aligned}$$

Then

$$\begin{aligned} &\frac{\partial^2 [\Delta V(\beta)]}{\partial \beta^2} \\ = \;& \int_{\theta_1}^{\theta_2} \frac{\partial^2 \Psi_1(y, \theta, \beta, s)}{\partial \beta^2} dF(\theta) + \int_{\theta_2}^{\overline{\theta}} \frac{\partial^2 \Psi_2(y, \theta, \beta, s, q)}{\partial \beta^2} dF(\theta) \\ & - \frac{\partial \Psi_1(y, \theta_1, \beta, s)}{\partial \theta_1} \frac{\partial \theta_1(y, \beta)}{\partial \beta} f(\theta_1) + \left(\frac{\partial \Psi_1(y, \theta_2, \beta, s)}{\partial \theta_2} - \frac{\partial \Psi_2(y, \theta, \beta, s, q)}{\partial \beta} \right) \frac{\partial \theta_2(y, \beta)}{\partial \beta} f(\theta_2) \\ & - \frac{\partial^2 \theta_1(y, \beta)}{\partial \beta^2} \Psi_1(y, \theta_1, \beta, s) f(\theta_1) + \frac{\partial^2 \theta_2(y, \beta)}{\partial \beta^2} [\Psi_1(y, \theta_2, \beta, s) - \Psi_2(y, \theta_2, \beta, s, q)] f(\theta_2) \end{aligned} \tag{115}$$

i) To sign the expression (115), first notice that $\frac{\partial \Psi_1(y,\theta,\beta,s)}{\partial \beta} = (1-\beta)\,\delta h'(x^*(\theta, \beta; s))\,\frac{\partial x^*(\theta,\beta;s)}{\partial \beta}$ (where we have used the ex post consumer's first-order condition) and

$$\begin{aligned} \frac{\partial^2 \Psi_1(y, \theta, \beta, s)}{\partial \beta^2} &= -\delta h'(x^*(\theta, \beta; s)) \frac{\partial x^*(\theta, \beta; s)}{\partial \beta} + (1-\beta)\,\delta h''(x^*(\theta, \beta; s)) \left(\frac{\partial x^*(\theta, \beta; s)}{\partial \beta} \right)^2 \\ &\quad + (1-\beta)\,\delta h'(x^*(\theta, \beta; s)) \frac{\partial^2 x^*(\theta, \beta; s)}{\partial \beta^2} \end{aligned}$$

Clearly, $\frac{\partial^2 \Psi_1(y,\theta,\beta,s)}{\partial \beta^2} > 0$, given (106) and (110).

ii) Second, notice that $\Psi_2(y,\theta,\beta,s,q)$ can reexpressed as

$$\begin{aligned}
&\Psi_2(y,\theta,\beta,s,q)\\
=\;&\theta\left[v\left(x^*(\theta,\beta;q)\right)-v\left(x^*(\theta,\beta;s)\right)\right]-\delta\left[h\left(x^*(\theta,\beta;q)\right)-h\left(x^*(\theta,\beta;s)\right)\right]\\
&-qx^*(\theta,\beta;q)+sx^*(\theta,\beta;s)+(q-s)\,y\\
=\;&\int_s^q\left\{\left[\theta v'\left(x^*(\theta,\beta;r)\right)-\delta h' x^*(\theta,\beta;r)-r\right]\frac{\partial x^*(\theta,\beta;r)}{\partial r}-x^*(\theta,\beta;r)+y\right\}dr\\
=\;&\int_s^q\left\{-(1-\beta)\,\delta h'\left(x^*(\theta,\beta;r)\right)\frac{\partial x^*(\theta,\beta;r)}{\partial r}-x^*(\theta,\beta;r)+y\right\}dr\\
=\;&\int_s^q\left\{-(1-\beta)\frac{\partial x^*(\theta,\beta;r)}{\partial\beta}-x^*(\theta,\beta;r)+y\right\}dr
\end{aligned}$$

Hence

$$\frac{\partial^2\Psi_2(y,\theta,\beta,s,q)}{\partial\beta^2}=\int_s^q\left\{\frac{\partial}{\partial\beta}\left[-(1-\beta)\frac{\partial^2 x^*(\theta,\beta;r)}{\partial\beta^2}\right]\right\}dr$$

Clearly, $\frac{\partial^2\Psi_2(y,\theta,\beta,s,q)}{\partial\beta^2}\geq 0$, given the assumption (112) .

iii) To sign the remaining terms, first note that $\frac{\partial\theta_1(y,\beta)}{\partial\beta}=\frac{\partial\theta_2(y,\beta)}{\partial\beta}=-\frac{\delta h'(y)}{v'(y)}$ and $\frac{\partial^2\theta_1(y,\beta)}{\partial\beta^2}=\frac{\partial^2\theta_2(y,\beta)}{\partial\beta^2}=0$ while $\frac{\partial x^*(\theta,\beta;r)}{\partial\theta}=-\frac{v'(x^*(\theta,\beta;s))}{\delta h'(x^*(\theta,\beta;s))}\frac{\partial x^*(\theta,\beta;s)}{\partial\beta}$. Using these results, it is easy to see that $-\frac{\partial^2\theta_1(y,\beta)}{\partial\beta^2}\Psi_1(y,\theta_1,\beta,s)\,f(\theta_1)=0$ and $\frac{\partial^2\theta_2(y,\beta)}{\partial\beta^2}\left[\Psi_1(y,\theta_2,\beta,s)-\Psi_2(y,\theta_2,\beta,s,q)\right]f(\theta_2)=0$. Furthermore, one can show that

$$\begin{aligned}
\frac{\partial\Psi_1(y,\theta_1,\beta,s)}{\partial\theta_1}&=(1-\beta)\,\delta h'(y)\frac{\partial x^*(\theta_1,\beta;r)}{\partial\theta_1}\frac{\partial\theta_1(y,\beta)}{\partial\beta}\\
&=(1-\beta)\,\delta h'(y)\frac{\partial x^*(\theta_1,\beta;s)}{\partial\beta}
\end{aligned}$$

and

$$\begin{aligned}
&\left(\frac{\partial\Psi_1(y,\theta_2,\beta,s)}{\partial\theta_2}-\frac{\partial\Psi_2(y,\theta,\beta,s,q)}{\partial\theta_2}\right)\frac{\partial\theta_2(y,\beta)}{\partial\beta}\\
=\;&(1-\beta)\,\delta h'(y)\frac{v'(y)}{\delta h'(y)}\frac{\partial x^*(\theta_2,\beta;q)}{\partial\beta}
\end{aligned}$$

Then we have

$$\begin{aligned}
\Psi_3\;\equiv\;&-\frac{\partial\Psi_1(y,\theta_1,\beta,s)}{\partial\theta_1}\frac{\partial\theta_1(y,\beta)}{\partial\beta}f(\theta_1)+\left(\frac{\partial\Psi_1(y,\theta_2,\beta,s)}{\partial\theta_2}-\frac{\partial\Psi_2(y,\theta,\beta,s,q)}{\partial\beta}\right)\frac{\partial\theta_2(y,\beta)}{\partial\beta}f(\theta_2)\\
=\;&(1-\beta)\,\delta h'(y)\left[\frac{\partial x^*(\theta_1,\beta;s)}{\partial\beta}f(\theta_2)-\frac{\partial x^*(\theta_1,\beta;s)}{\partial\beta}f(\theta_1)\right]\\
=\;&(1-\beta)\left[\delta h'(y)\right]^2\left[\frac{f(\theta_2)}{\theta_2 v''(y)-\beta h''(y)}-\frac{f(\theta_1)}{\theta_2 v''(y)-\beta h''(y)}\right]
\end{aligned}$$

Hence, $\Psi_3\geq 0$ given the assumption (111) (sufficient condition).

iv) Finally, it follows from items i), ii) and iii), that $\frac{\partial^2[\Delta V(\beta)]}{\partial\beta^2} > 0$.

B) Next consider two distributions of self-control problems, $M_I(\beta)$ and $\widehat{M}_I(\beta)$. Both distributions have the same mean

$$\int_0^1 \beta dM_I(\beta) = \int_0^1 \beta d\widehat{M}_I(\beta) \Leftrightarrow \int_0^1 \left[\widehat{M}_I(\beta) - M_I(\beta)\right] d\beta = 0$$

but $\widehat{M}_I(\beta)$ is more dispersed, in the sense of second-order stochastic dominance

$$\int_0^\beta \left[\widehat{M}_I\left(\widetilde{\beta}\right) - M_I\left(\widetilde{\beta}\right)\right] d\widetilde{\beta} \geq 0 \text{ for } \beta \in [0,1]$$

with a strict inequality for a set of values of β with a positive probability. Then

$$\begin{aligned} & \int_0^1 \Delta V_I(\beta)\, d\widehat{M}_I(\beta) - \int_0^1 \Delta V_I(\beta)\, dM_I(\beta) \\ = & \int_0^1 \left\{ \frac{\partial^2 [\Delta V_I(\beta)]}{\partial \beta^2} \int_0^\beta \left[\widehat{M}_I\left(\widetilde{\beta}\right) - M_I\left(\widetilde{\beta}\right)\right] d\widetilde{\beta} \right\} d\beta > 0 \end{aligned} \tag{116}$$

(where the latter form is derived through integration by parts). Since the inequality (116) holds in each group I, a similar inequality holds at the aggregate level:

$$\int \left[\int_0^1 \Delta V_I(\beta)\, d\widehat{M}_I(\beta)\right] dG(I) > \int \left[\int_0^1 \Delta V_I(\beta)\, dM_I(\beta)\right] dG(I)$$

(where $G(I)$ is the cumulative distribution function of consumer groups). In words, the private benefits from the reform are larger under the more dispersed distribution of self-control problems.

The distribution of self-control problems and social benefits from introducing sin licenses

Our second result allows us to directly assess how the social benefits of the reform depend on the dispersion of self-control problems. We show that, under certain conditions, social benefits (ΔW) from introducing sin licenses are larger when consumers differ more in terms of self-control problems.

To be more specific, let $\beta_I^L = \left(1 - \tau_2 \left[\delta_I h_I'\left(x_I^o\left(\underline{\theta}_I\right)\right)\right]^{-1}\right)$ and $\beta_I^H = \left(1 - \tau \left[\delta h_I'\left(x_I^o\left(\overline{\theta}_I\right)\right)\right]^{-1}\right)$, where clearly $\beta_I^L < \beta_I^H$; we also assume that τ is such that $\beta_I^L > 0$. Let us consider two distributions of self-control problems, $M_I(\beta)$ and $\widehat{M}_I(\beta)$, where $\widehat{M}_I(\beta)$ is constructed from $M_I(\beta)$ using two transformations. i) In the left tail of the distribution ($\beta \leq \beta_I^L$) we shift frequency mass from higher values of β towards lower values of β. ii) At the high end of the distribution, we shift frequency mass from types with mild self-control problems $\beta \in [\beta_I^H, 1)$ to the rational type $\beta = 1$. Then the social benefits from the reform (ΔW) are larger (or at least less negative) under the more dispersed distribution $\widehat{M}_I(\beta)$.

Proving the result. Let us define a (shadow) tax rate

$$\tau_I^*(\beta,\theta) = (1-\beta)\,\delta_I h_I'(x_I^o(\theta))$$

that would induce optimal consumption $(x_I^o(\theta))$ for ex post consumer type (I,β,θ).

A) Now suppose that for some ex ante types $\beta \leq \beta_I^L$ we have

$$\tau_I^*(\beta,\theta) \geq \tau_2 > \tau \tag{117}$$

for all θ. Here the threshold value β_I^L is given by

$$\tau_I^*\left(\beta_I^L, \underline{\theta}_I\right) = \tau_2 \Longleftrightarrow \beta_I^L = 1 - \frac{\tau_2}{\delta_I h_I'\left(x_I^o\left(\underline{\theta}_I\right)\right)}$$

(where $\underline{\theta}_I$ is minimum shock realization in group I). Then with any shock realization θ, consumer types $\beta \leq \beta_I^L$ consume more than (or no less than) the rational/optimal amount $x_I^o(\theta)$, both under sin taxes and under sin licenses. However, the higher price $q = 1+\tau_2$ under the system of sin licenses helps these consumers to get closer to the optimal/rational consumption level $x_I^o(\theta)$. Hence introducing sin licenses improves the welfare of types $\beta \leq \beta_I^L$, from the social point of view: more formally $\Delta W_I(\beta) > 0$ for $\beta \leq \beta_I^L$. Moreover, within this subgroup ($\beta \leq \beta_I^L$), licenses are more beneficial for more irrational consumers, $\partial \Delta W_I(\beta)/\partial\beta$. Using the notation introduced in the proof of Proposition 3

$$\begin{aligned}\Delta W_I(\beta) &= \int_{\theta_1}^{\theta_2} \left[\widehat{u}^o(y,\theta) - \widehat{u}^o(x^*(\theta,\beta;s),\theta)\right] dF(\theta) \\ &\quad + \int_{\theta_1}^{\theta_2} \left[\widehat{u}^o(x^*(\theta,\beta;q),\theta) - \widehat{u}^o(x^*(\theta,\beta;s),\theta)\right] dF(\theta)\end{aligned}$$

(To simplify notation, we leave out the group subindex I). Next note that

$$\begin{aligned}& \widehat{u}^o(x^*(\theta,\beta;q),\theta) - \widehat{u}^o(x^*(\theta,\beta;s),\theta) \\ =& \int_s^q \left[\frac{\partial \widehat{u}^o(x^*(\theta,\beta;q),\theta)}{\partial x^*}\frac{\partial x^*(\theta,\beta;r)}{\partial r}\right] dr \\ =& -\int_s^q \left[(1-\beta)\,\delta h'(x^*(\theta,\beta;r)) - (r-1)\right] \frac{\partial x^*(\theta,\beta;r)}{\partial r} dr\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial\left[\Delta W_I(\beta)\right]}{\partial\beta} &= \left\{\int_{\theta_1}^{\theta_2}\left[\theta v'(y)-\delta h'(y)-1\right]dF(\theta)\right\}\frac{dy}{d\beta} \\
&\quad+\int_{\theta_1}^{\theta_2}\left[(1-\beta)\,\delta h'\left(x^*(\theta,\beta;s)\right)-\tau\right]\frac{\partial x^*(\theta,\beta;s)}{\partial\beta}dF(\theta) \\
&\quad+\int_{\theta_2}^{\overline{\theta}}\left[\int_s^q \delta h'\left(x^*(\theta,\beta;r)\right)\frac{\partial x^*(\theta,\beta;r)}{\partial r}dr\right]dF(\theta) \qquad (118)\\
&\quad-\int_{\theta_2}^{\overline{\theta}}\left[\int_s^q\left[(1-\beta)\,\delta h''\left(x^*(\theta,\beta;r)\right)\right]\frac{\partial x^*(\theta,\beta;r)}{\partial r}\frac{\partial x^*(\theta,\beta;r)}{\partial\beta}dr\right]dF(\theta) \\
&\quad-\int_{\theta_2}^{\overline{\theta}}\left[\int_s^q\left[(1-\beta)\,\delta h'\left(x^*(\theta,\beta;r)\right)-(r-1)\right]\frac{\partial^2 x^*(\theta,\beta;r)}{\partial r\partial\beta}dr\right]dF(\theta) \\
&< 0
\end{aligned}$$

When signing the derivative (118), we have used to the following facts: 1) The term on the first row <0, since 1a) by the first-order condition (87)

$$\int_{\theta_1}^{\theta_2}\left[\theta v'(y)-\delta h'(y)-1\right]dF(\theta) = -\tau\left[F(\theta_2)-F(\theta_1)\right]-(q-s)\left[1-F(\theta_2)\right]<0$$

and 1b) $\frac{dy}{d\beta}\geq 0$, by (92) 2) The second row <0, since 2a) by (117)

$$(1-\beta)\,\delta h'\left(x^*(\theta,\beta;s)\right)>\tau_I^*(\beta,\theta)>\tau\Leftrightarrow(1-\beta)\,\delta h'\left(x^*(\theta,\beta;s)\right)-\tau>0$$

and 2b) $\frac{\partial x^*(\theta,\beta;s)}{\partial\beta}<0$, by (106). 3) The third row <0, since $\frac{\partial x^*(\theta,\beta;r)}{\partial r}<0$ by (105). 4) The fourth row <0, since $\frac{\partial x^*(\theta,\beta;r)}{\partial r}<0$ and $\frac{\partial x^*(\theta,\beta;s)}{\partial\beta}<0$ by (105) and (106). 5) The fifth row <0, since 5a) by (117)

$$\begin{aligned}
(1-\beta)\,\delta h'\left(x^*(\theta,\beta;r)\right) &> \tau_I^*(\beta,\theta)>r-1\Leftrightarrow \\
(1-\beta)\,\delta h'\left(x^*(\theta,\beta;r)\right)-(r-1) &> 0
\end{aligned}$$

(where $(r-1)\in[\tau,\tau_2]$, since $r\in[s,q]$) and 5b) $\frac{\partial^2 x^*(\theta,\beta;r)}{\partial r\partial\beta}>0$ by (109).

B) Next suppose that for some ex ante types $\beta\geq\beta_I^H$ we have

$$\tau_I^*(\beta,\theta)\leq\tau<\tau_2 \qquad (119)$$

for all θ. Here the threshold value β_I^H is given by

$$\tau_I^*\left(\beta_I^H,\overline{\theta}_I\right)=\tau\Longleftrightarrow\beta_I^H=1-\frac{\tau}{\delta h_I'\left(x_I^o\left(\overline{\theta}_I\right)\right)}$$

(where $\overline{\theta}_I$ is maximum preference shock realization in group I). Then with any shock realization θ, consumer types $\beta \geq \beta_I^H$ consume less than (or no more than) the rational/optimal amount $x_I^o(\theta)$, both under sin taxes and under sin licenses. However, a possibly binding quota y and the higher price $q = 1 + \tau_2$ under the system of sin licenses may pull realized consumption still further away from optimal/rational consumption $x_I^o(\theta)$. Hence introducing sin licenses cannot improve the welfare of types $\beta \geq \beta_I^H$, from the social point of view, and it may lower welfare. More precisely, and more formally

$$\Delta W_I(\beta) \leq 0 \text{ for } \beta \in [\beta_I^H, 1) \tag{120}$$

where strict inequality holds, if a nearly rational type $\left(\beta \in [\beta_I^H, 1)\right)$ chooses a quota y that binds for some θ. On the other hand we also know that the rational type never chooses a binding quota y, and hence introducing a system of sin licenses ($p = s,\ q > s$) does not affect the choices or the welfare of the rational type:

$$\Delta W_I(\beta) = 0 \text{ for } \beta = 1 \tag{121}$$

C) The change in social welfare in group I, following the introduction of sin licenses, is given by $\Delta W_I = E_\beta[\Delta W_I(\beta)] = \int \Delta W_I(\beta)\, dM_I(\beta)$, where the expectation is taken with respect to the distribution $M_I(\beta)$. Next, starting from $M_I(\beta)$, we construct a new, more dispersed distribution, say $\widehat{M}_I(\beta)$, in the following way: i) In the left tail of the distribution ($\beta \leq \beta_I^L$) we shift frequency mass from higher values of β towards lower values of β. ii) At the opposite end of the distribution, we shift frequency mass from types with mild self-control problems $\beta \in [\beta_I^H, 1)$ to the rational type $\beta = 1$. iii) Furthermore, *if* we want that $\widehat{M}_I(\beta)$ is a mean-preserving spread of $M_I(\beta)$, we further require that steps i) and ii) are carried out in such a way that both distributions have the same mean, $\int \beta d\widehat{M}(\beta) = \int \beta dM(\beta)$.

Now, given the properties (118), (120) and (121) established above, it is clear that

$$\int \Delta W_I(\beta)\, d\widehat{M}_I(\beta) > \int \Delta W_I(\beta)\, dM_I(\beta) \tag{122}$$

Finally moving from the group level to the aggregate level is easy. Since the inequality (122) holds in each group I, a similar inequality holds at the aggregate level:

$$\int \left[\int \Delta W_I(\beta)\, d\widehat{M}_I(\beta)\right] dG(I) > \int \left[\int \Delta W_I(\beta)\, dM_I(\beta)\right] dG(I)$$

(where $G(I)$ is the cumulative distribution function of consumer groups). In words, sin licenses are more likely to be socially beneficial under the more dispersed distribution of self-control problems.

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